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BIOMETRIKA.

ON THE RESULT OF CROSSING JAPANESE WALTZING WITH ALBINO MICE.

BY A. D. DARBISHIRE, Balliol College, Oxford.

THE object for which the following experiments were undertaken was to test the validity of Mendel's Principles of Heredity, which have recently acquired considerable importance, by becoming intimately involved in the question of the origin of species. The term Mendelian Principles is used in its widest sense, to include not merely the simple Mendelian phenomena of Dominance or Segregation, but the much more fundamental doctrine of gametic purity.

METHOD OF DESCRIPTION.

The method of description employed in this paper is the same as that used in the three reports on these experiments which have already appeared: the mice are classified, according to the relative extent of the coloured patches on a white ground, into six groups.

Group 1 (Fig. 1) has more white and less extent of coloured patches than the normal waltzing mouse. The distribution of colour on a waltzer is shown in Fig. 6.

I wish to draw special attention to the existence of this group, because Castle*, who quoted my second paper for another purpose, denies that any mice with less colour than a waltzer had occurred in my experiments

Group 2 (Fig. 2) has about as much white as a normal waltzing mouse.

Group 3 (Fig. 3) has less white (i.e. greater extent of coloured patches) than a waltzer.

Group 4 (Fig. 4) has still less white, and leads on to

Group 5 (Fig. 5) which has no pure white, but the belly is whitish, not grey.

Group 6 contains mice whose bellies are nearly the same colours as their backs.

* "Mendel's Law of Heredity," *Proc. American Academy*, Jan. 1903, pp. 533—48.

The individuals of each group can also be classified according to the colour of the coloured patches.

Class *a* = yellow.

Class *b* = fawn yellow.

Class *c* = pale wild colour
Class *d* = dark wild colour } , i.e. that of the house-mouse.

Class *e* = black.

Class *f* = "lilac" = pale blue grey; at present only exhibited by the offspring of hybrids.

Class *g* = chocolate.

The letter "*p*" after a mouse indicates that it has a pink eye, the letter "*W*" that it waltzes; to take two examples, 2*b* (*pW*) is a waltzer, 6*d* or 6*c* is a house-mouse.

I propose to defer a detailed discussion of the nature of these categories to a later date: but I hope the foregoing remarks will leave no doubt in the reader's mind as to what is meant by each class; in fact, all the classes except the first two explain themselves. I am sorry the word fawn was used in my second paper, because it is used to denote both *a* and *b* by mouse fanciers; *a* is simply a yellow colour; *b* is yellow, with, in many cases, black in it; at any rate, there are intermediate stages linking *b* with *c* (i.e. pale wild), whereas *a* is not so linked with *c* except through *b*.

In cases of intermediate colours the mice are entered 5*bc* and so forth: a minute discussion of the skins (which are kept) is deferred.

Methods of Registering.

The method of keeping, registering and cataloguing which I now pursue has gradually grown up of itself in the course of a year rather as a result of necessity than by deliberate invention on my part. Lack of a proper method of registering at the beginning of the experiments rendered many of the dealings with mice a matter of great difficulty; and I suggest that if the reader is taking up work of this kind he will not suffer from a similar inconvenience if he adopts a method similar in principle to that which I am about to describe.

During the latter part of the experiment there were some fifteen hundred mice in the room set apart for the purpose; and it was quite a simple matter to look up in the catalogue book any mouse in the room, and *vice versâ*, to find in the room a mouse of any given ancestry. The method is very simple, and was based on a system of cross-reference, which enabled one to refer (as already stated) from the books to the mice, and from the mice to the books.

The following account, then, will be of little if any interest to the reader who does not intend to do similar work.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

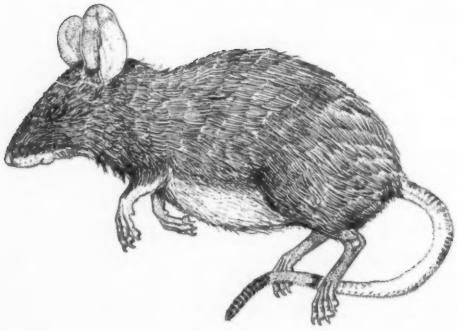


Fig. 5.

Fig. 6.

Let us imagine that our object is to make crosses between albino and Japanese waltzing mice, and that we are provided with a stock of waltzing mice which we know to breed true, and of albino mice of authentic pedigree. Two sorts of books are necessary—(i) a book recording the matings, and later on (ii) an illustrated catalogue. The entry of the cross in the mating book is quite simple. The date underneath the description refers to the day on which the mice were put together; and the occurrences which come to pass on the subsequent dates are also recorded.

Cross 1.

♀ Albino (pure bred) from Mr Steer.

♂ Japanese waltzer.

Dec. 24.

♂ removed, Jan. 10th.

8 young born, Jan. 16th.

♀ removed, Feb. 8th.

Young sexed, Feb. 25th.

The offspring are catalogued in two places for reasons which will shortly be explained: first they are catalogued as soon as the colour of their eyes and coat is discernible; this might quite well be done in the mating book; but I find it more convenient to have a separate book (First Catalogue) for the purpose: the coat-colour and eye-colour of each mouse are entered directly they can be plainly seen, so that a minimum number of records is lost through cannibalism or disease: nothing else is recorded; the sex is not (for it cannot be) determined. Such an entry appears as follows:—

Cross 1. 5*c*, 5*c*, 5*c*, 3*c*, 2*c*, 2*c*, 1*c* (ix. 21).

The entry in brackets gives the date on which the mice were catalogued; if this is not done and a note is made in the mating book that a young one of a certain description has escaped or died, doubt would arise in the mind of the observer as to whether that mouse had been catalogued or not.

When the young are sexed they are entered in the illustrated catalogue, which, beside the record of matings (already described), is the second kind of book necessary for the observations. In this book each mouse is given a name: thus the first mouse which happens to be described, in the litter of the imaginary cross which we entered in the mating book above, is $1^{a\,i}$, the second $1^{a\,ii}$, and so on. The "a" signifies that the individual belonged to the first litter produced by those parents; individuals of the second litter produced by the same parents would receive the names $1^{b\,i}$, $1^{b\,ii}$, $1^{b\,iii}$, etc. Roman numerals are used to indicate the individuals of the family because they are never likely to become too large to be convenient, for litters very rarely exceed, and, in fact, very seldom attain, the number of ten, and $1^{a\,x}$ is quite convenient to write. In cases of albinos and self-coloured mice, in which the tail is uniformly covered with pigment, I have merely written the description of the mouse, as a picture of such a one would

be superfluous: but in all cases of piebald mice I have made a drawing of the animal as seen from the right, left, and dorsal aspects by painting the colour-pattern on outline drawings of mice (printed for me by Messrs Palmer and Howe of Manchester) kept for the purpose; and in cases of completely coloured mice with incompletely coloured tails I have merely drawn one view to show the arrangement of colour on the tail. All mice, which have reached the age at which they are sexed, have been described in this manner.

Now suppose that one wants to pair these hybrids (First Crosses) together; one proceeds in this way.

Another mating book has to be used to enter the matings of the hybrids (to avoid hopeless confusion); and the numeral which indicates a cross or pair is preceded by some letter (such as *H*) to distinguish it from the same number which refers to a cross between a waltzing mouse and albino. I have called these pairs or crosses H_1 H_2, etc. The individuals paired are entered thus in the mating book:

H_1
♀ 7^{aiii}
♂ 1^{ai} *March* 9.

Against 7^{aiii} in the illustrated catalogue book H_1 is written, so that one is enabled to refer from the ancestry of a mouse to its place in the room (for of course all pairs of the same kind, i.e. *H*'s for example, are kept together). Whenever 7^{aiii} is mated again to form another pair or cross, the number of each successive pair in which it is concerned is put after it, so that if one wants to find 7^{aiii} one looks at the last number against its name in the mating book.

The value of the illustrated catalogue lies in the fact that in many families the brothers and sisters are so unlike that when they are again looked up reference to the catalogue will convince one that one is dealing with the right mouse: of course they are identified by the number on their cage, so that their picture is not a necessary means of identification, but it is an additional safeguard against error.

Fig. 7 represents a sample page of the illustrated catalogue on a reduced scale.

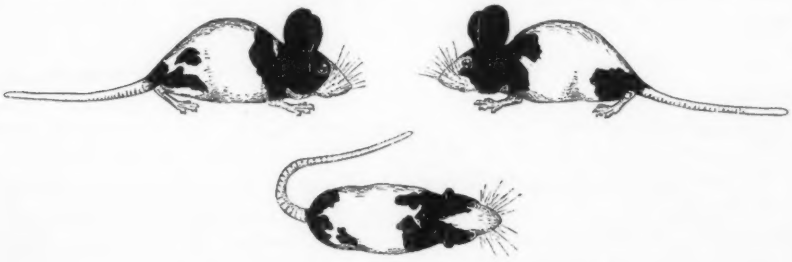


Fig. 7.

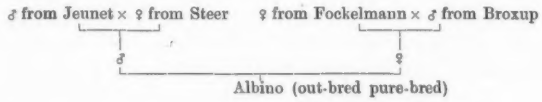
Description of Parents.

Japanese Waltzing Mice.

These animals are distinguished from ordinary (say albino) fancy mice by their smaller size, by their extreme liability to disease, but especially by their curious habit of spinning round, by the ceaseless movement of their heads, and by the retrograde motion of the body when not waltzing. The mice used by me had pink eyes, and bore patches on the body of colour indicated by *b*: the range of variability in colour was practically nothing; that of the extent of the coloured patch was small: accurate pictures have not been kept of all of them because the simple Mendelian statements, which it was the original object of the experiment to test, did not lead to the expectation that minute study of these mice would be necessary; the exact distribution of the colour on the waltzing parent is therefore not known in all cases; in those cases in which pictures of the waltzers were made, a classification of them has been made into (α) waltzers with less colour than Fig. 7, and (β) waltzers with more colour than Fig. 7. All the waltzers used were of pure strain.

Albino Mice.

In the previous experiments of this kind (conducted by von Guaita) the albinos used were of uniform ancestry, being in-bred for 29 generations; that is, they were also pure-bred. I obtained my albino stock chiefly from Mr Steer in England, from M. Jeunet in Paris, and from Herr Fockelmann in Hamburg. From the pure-bred mice which I got from Mr Steer I raised in-bred pure-bred albinos, and by mating the foreign mice with English ones I obtained out-bred pure-bred mice. An out-bred pure-bred mouse would have this ancestry for example:



Out-bred cross-bred mice were obtained by crossing black or yellow mice with albinos—in-bred cross-bred were obtained by waiting for a litter, from such a cross, in which there were both albinos and blacks (or yellows), then pairing a black mouse with his white sister, and so on for many generations.

A list of all the mice bred during the experiment is given in Appendix I., Tables A to H.

The Hybrids.

Before describing the results of crossing albinos and Japanese waltzers, I think it is advisable to define clearly the terms which I shall use to describe each generation. The offspring produced by crossing a Japanese waltzer with an albino I propose to call the hybrid, simply; the offspring of these hybrids paired together,

children of hybrids; the offspring of the hybrids crossed with albinos, children of hybrids and albino. These terms are perfectly simple, but it is impossible to be too explicit about terms such as these.

The hybrids are invariably dark-eyed; their coat is never completely white, and they never waltz; the two facts about them which strike one immediately are (i) that whereas the parents are all like the other members of their race, the hybrids are by no means all alike; they exhibit much variability in coat-colour and in the distribution of their colour; (ii) that not only do they differ among themselves, but they are absolutely different from either parent; the character of neither parent can be said to be dominant over the other in the sense that it exists in the hybrid to the exclusion of the other, which is what Mendel meant by dominance. To make up, however, for this deficiency in dominance, the characters of both parents are recessive, in so far that neither of them appear in the hybrid. This is plainly not a case of simple Mendelian dominance, though I am perfectly ready to admit that it may be an instance of the mysterious properties of heterozygotes, confessing at the same time that I do not see how the admission of this fact helps. It seems to me that we have not got any further in this direction than Darwin had when he called phenomena of this kind reversions to ancestral condition.

The eye-colour and the amount and nature of coat-colour are not the only new characters which appear in the hybrids: there is a correlation between the size of the patches of colour and the intensity of the colour in such a way that the lighter the colour the more space it occupies. The measure of this correlation is between $-0{\cdot}2$ and $-0{\cdot}3$. This character of the hybrids is clearly absent in the parents; for, while the waltzers vary slightly in the amount of colour which they possess, the colour itself does not vary: the albinos have no colour at all*.

We have up to now merely stated the fact that the hybrids are variable; it remains to try to account for this variability. How Mendelians will account for it I do not know, but I have facts to show that it is due to ancestry of the parent forms.

Fraternal correlation.

From the Mendelian point of view the families should be all similar; and this would lead to fraternal correlation $=0$. Fraternal correlations are, however, remarkably like those observed in fraternities which are not cross-bred; taking either the colour of the coat or the extent of the white patches as the character to be examined, we find a fraternal correlation very nearly equal to $0{\cdot}5$. This shows that there is (1) a sensible range of variation within each family, and (2) a considerable range in the mean characters of different families. The variation within the family seems to me incompatible with any Mendelian hypothesis which does

* The obvious want of "normality" in the distribution of coat-colour and of whiteness in all the generations observed renders the determination of the correlation coefficients very difficult. The process adopted, which is due to Professor Pearson, will be explained and justified in a future paper by him.

not involve a complexity in the constitution of the waltzing parent, leading to the assumption that its coat-colour depends not on one allelomorph, but on a number. This might be conceivable, but it would involve a reduction in the proportion of albino young among children of hybrids incompatible with that observed in the next generation.

We see therefore that characters of hybrids are not compatible with, still less explained by, Mendelian principles; it still remains to account for them.

Parental correlation.

If we classify the hybrids into those in which the albino parent was pure-bred, and those in which it was cross-bred, i.e. in which it had some pigmented ancestors, and determine the correlation between the colour of the young and the purity of the albino parent, we find that it is about $-0{\cdot}15$, and the correlation between the amount of whiteness in the hybrids and purity of albino parent about $-0{\cdot}2$. These are quite sensible correlations; they show that there is a distinct relation between the pigmentation of the hybrids and of the ancestors of their albino parent: they show therefore that the albino cannot be said to be gametically pure in respect of its whiteness, but that in order to predict the character of its young a knowledge of its ancestry is necessary. The character of the hybrids is therefore influenced by the ancestry of the albino parent: but it is also affected by the character of the waltzing parent. As has already been stated, pictures of all the waltzing parents of hybrids have not been made, but the waltzing parents of 210 out of the 340 hybrids obtained are represented by accurate diagrams, and these have been divided into (α) those with less extent of coloured fur than the mouse figured in Fig. 1, and (β) those with more; from waltzers of category (α) 113 young have been produced; and from (β) 97. The coat-colour and degree of whiteness in these young is shown in Tables IV. and V. (page 38). It will be seen that there is considerable correlation between the amount of white in the waltzing parent and that in the resultant hybrid. Table V. shows the correlation between coat-colour of hybrids and whiteness in their waltzing parent. In both these characters the coefficient of parental correlation is very nearly $\frac{1}{2}$.

The amount and nature of the colour of a hybrid are therefore not constant, but exhibit great variability. (a) The hybrid does not show dominance of the character of either parent, and (b) its variability is sensibly correlated, not only with the actual character of the "dominant" parent, but also with the character of the ancestry of the "recessive" albino parent. Whatever theory these facts are to be reconciled with, I maintain that it is *not* with the theory of gametic purity. How far they are reconcilable with other theories of heredity is a more difficult question.

In order to find out whether the results obtained are in accordance with the law of ancestral inheritance, as stated by Pearson (cf. *Biometrika*, Vol. II. pp. 211–229), it would be necessary to examine more closely than is here possible the effect produced upon all the coefficients of parental, grandparental, and other ancestral correlation, and upon the paternal correlation by the rigid selection of

parents which has taken place in this as in all other hybridising experiments. These results are extremely complex, and cannot be predicted without long and difficult enquiry; some of them have, however, been examined by Pearson ("Mathematical Contributions to the Theory of Evolution," *Phil. Trans.*, Vol. 200 A, 1902), who shows that "sexual and natural selection can sensibly modify the intensity of inheritance as measured by the coefficient of correlation; the former tends to raise, the latter to lower its intensity" (p. 43).

The original statement embodied by Galton and Pearson in the Law of Ancestral Heredity relates to a population in which two conditions hold, namely, (i) all mating is at random; in other words the correlation between the characters of male and female mates in any generation is 0, and (ii) all the individuals in the generation have an equal chance of producing young. Now it is a fact, which is often forgotten, and on which I wish to lay great emphasis, that in the experiments described, and in all cases where two races or breeds are systematically crossed, the first of these conditions is deliberately destroyed, and a process of assortative mating is performed, such that the coefficient of correlation between parents is given the value -1 instead of zero. Thus, in the present case, if we know that the father of the hybrid is an albino, we may certainly infer that the mother is not, and *vice versâ*. The effect of this perfect negative correlation between the parents must be to reduce the coefficient of correlation between parents and offspring to 0 (i.e. to destroy the correlation altogether) in every case in which the results of reciprocal crosses are identical.

This may be readily seen by a simple illustration: if we call waltzers with more colour β, and with less α, and the albino parents A, and if we write the female first, the possible unions are:

$$\alpha A,$$
$$\beta A,$$
$$A\alpha,$$
$$A\beta.$$

And if reciprocal crosses produce the same results, the offspring of αA and $A\alpha$ will be identical, as will those of βA and $A\beta$. Now suppose that the pairs αA or $A\alpha$ each produce a offspring of one kind, b of a second kind, while the pairs βA or $A\beta$ produce c young of the first kind and d of the second: if we make a table of the correlation between (say) maternal character and character of young, it will have the form:

Young.

	1	*2*	Totals
α or β	$a+c$	$b+d$	$(a+b+c+d)$
A	$a+c$	$b+d$	$(a+b+c+d)$
Totals	$2(a+c)$	$2(b+d)$	$2(a+b+c+d)$

And since every measure of association or correlation becomes zero with the difference of the products of diagonally opposite squares of such a table, we see that whatever measure be adopted must in this case give a value = 0, since the difference of the cross products = $(a+c)(b+d)-(a+c)(b+d)=0$.

It is of course evident that this important result is a necessary arithmetical consequence of the way in which the parents have been mated and of the identity between reciprocal crosses; it does not depend on any theory of the nature of inheritance, and it involves no statement about the mean character of the young produced; the young may be intermediate between their two parents, or exactly like one or the other, or wholly unlike both.

A further effect of breeding experiments, which involve sudden (and generally great) changes in the correlations between male and female, is a condition of instability during the first few generations. A race may become stable with any constant amount of assortative mating, but a sudden change results in a condition of instability, making it peculiarly difficult to discuss those first generations which are too often the only experimental data available.

These considerations, usually forgotten by those who criticise the law of ancestral inheritance, must be carefully borne in mind by anyone who wishes to realise the questions actually at issue.

We have seen that a process of crossing, such as that performed with the mice described, involves a total absence of correlation between parents of either sex and offspring if the individuals of either race are chosen with equal frequency as male and as female parents. It must be pointed out that many of the tables which accompany this paper are not tables showing correlation between all parents of one *sex* and their offspring; but tables of correlation between all parents of one *character*, whatever their sex, and their young.

An examination of the full list of hybrids given in Table A, Appendix I., will show that the results of reciprocal crosses are so nearly identical as to justify the neglect of sex in the treatment of the data: a table of the correlation between the coloured waltzing mice or the albino mice and their hybrid young such as any of the Tables IV.—VII., may therefore be considered to give approximately the same result as if each referred to half the whole crosses from random unions between waltzing mice and albinos—that half namely in which all the parents are of one sex.

Such a way of regarding the tables is at best only an approximation to the truth; a fuller discussion of the parental and indeed of all correlations observed during the experiments must be published later: in the meantime the present way of regarding them may be useful as a preliminary test of their compatibility with the law of ancestral inheritance.

On the view suggested Tables VI. and VII. may be regarded as showing the result of two operations; it may be considered that first one parent has been

rigidly selected and that secondly the coefficient of assortative mating between this parent and the other has been made equal to -1. By this supposition we are enabled to apply a formula provided by Pearson (*Phil. Trans.* Vol. 200 A, 1902, pp. 39—40).

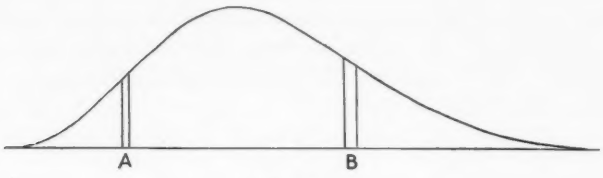


Fig. 8.

If we could make a diagram showing the frequency of any character, coat-colour or other, among mice in general, we may assume that the result would be something like Fig. 8; and the albino mice would be represented in relative frequency and position by one small strip (say A) of such a diagram, the waltzing mice by another small strip (say B).

Before the experiment began each set of mice, albinos and waltzers, formed a separate community, the individuals mating together, either at random or not, and having a standard deviation different from that of mice in general; the assortative mating within these groups we may call r_{12}; and let us call the ratio between the standard deviation of albinos and that of mice in general μ_1, the ratio between that of waltzers and that of mice in general μ_2.

Now calling r_{13}, r_{23} the coefficients of correlation between normal male and female mice and their young, and calling σ_3 the standard deviation of filial mice in general resulting from unselected unions, we have to find the relations between these quantities and Σ_3, R_{13}, the standard deviation and parental correlation of the filial mice resulting from a cross, in which assortative mating has been carried so far that the coefficient of correlation between parents, which we will call ρ_{12}, is equal to -1.

In the memoir quoted, Pearson has shown* that these quantities are connected by the following relations:

Writing for brevity

$$\beta_{13} = \frac{r_{13} - r_{23}\, r_{12}}{1 - r_{12}^{\,2}}$$

and

$$\beta_{23} = \frac{r_{23} - r_{13}\, r_{12}}{1 - r_{12}^{\,2}},$$

* Pearson's investigation is not strictly applicable to the present case, because he does not consider the long-continued selection of parents in past generations involved in the statement that the races used are pure.

we have

$$\Sigma_3^2 = \sigma_3^2 \{1 - \beta_{13}^2 (1 - \mu_1^2) - \beta_{23}^2 (1 - \mu_2^2) - 2 (r_{12} - \beta_{12} \mu_1 \mu_2) \beta_2 \beta_3\},$$

$$\Sigma_3 R_{13} = \sigma_3 \{r_{13} - (1 - \mu_1) \beta_{13} - (r_{12} - \rho_{12} \mu_2) \beta_{23}\}.$$

Now we are proceeding on the assumption that in the experiments under discussion the two coefficients of parental correlation r_{13} and r_{23} are equal; we may therefore suppress their suffixes and write each $= r$; further, we may express the purity of the original races used by writing for each of them r_{12}, the coefficient of assortative mating, $= 1$.

We have then

$$\beta_{23} = \beta_{13} = \frac{r(1 - r_{12})}{1 - r_{12}^2} = \frac{r}{1 + r_{12}} = \frac{r}{2};$$

and our formulae become

$$\Sigma_3^2 = \sigma_3^2 \left\{1 - \frac{r^2}{4}(1 - \mu_1^2) - \frac{r^2}{4}(1 - \mu_2^2) - \frac{2r^2}{4}(1 - \rho_{12} \mu_1 \mu_2)\right\}$$

and

$$\Sigma_3 R_{13} = \sigma_3 \left\{r - \frac{r(1 - \mu_1)}{2} - \frac{r(1 - \rho_{12} \mu_2)}{2}\right\}$$

$$= \sigma_3 \left\{\frac{r\mu_1}{2} + \frac{r\rho_{12} \mu_2}{2}\right\}.$$

Now suppose the suffix 1 to denote the parent whose correlations we are investigating and let us first examine the waltzing parent, so that the suffix 2 denotes an albino; we may fairly say that the ratio of the standard deviation of coat-colour among albinos to that of mice in general $= 0$, and we have for R_{13} the correlation between waltzing mice and their hybrid offspring,

$$R_{13} = \frac{\frac{r\mu_1}{2}}{\sqrt{1 - \frac{r^2}{4}(1 - \mu_1^2) - \frac{r^2}{4} - \frac{2r^2}{4}}},$$

$$R_{13} = \frac{\frac{r\mu_1}{2}}{\sqrt{1 - r^2 + \frac{r^2 \mu_1^2}{4}}},$$

and we see that the value to be expected depends (i) on the correlations between parent and offspring in those simpler cases of random mating first dealt with by Galton and Pearson in the Law of Ancestral Heredity, and (ii) on μ_1, the ratio between the standard deviation of the selected race of waltzing mice and that of the general population from which they are descended. We have no means of evaluating μ_1 in the case of coat-colour, but we know that it is small. We may

fairly assume for r the value so often found by Pearson and his pupils, and we will therefore write $r = \frac{1}{2}$; so that we have

$$R_{13} = \frac{\frac{1}{4}\mu_1}{\sqrt{\frac{3}{4} + \frac{\mu_1^2}{16}}},$$

which may clearly take a great number of values according to the value of μ_1 ranging from 0 when $\mu_1 = 0$ to 1 when $\mu_1 = \infty$.

We have found the correlation between a variable group of selected parents, the mates being invariable. To find the correlation between these invariable mates and their offspring we must write $\mu_1 = 0$; and we have, remembering that $\rho_{12} = -1$,

$$\Sigma_3 R_{13} = \sigma_3 \left\{ r - \frac{r}{2} - \frac{r}{2} + \frac{r}{2}\rho_{12}\mu_2 \right\},$$

or

$$\Sigma_3 R_{13} = \sigma_3 \left\{ \frac{r\rho_{12}\mu_2}{2} \right\} = -\frac{\sigma_3 r \mu_2}{2},$$

and

$$\Sigma_3^2 = \sigma_3^2 \left\{ 1 - r^2 + \frac{r^2\mu_2^2}{4} \right\},$$

or

$$R_{13} = \frac{-\frac{1}{2}r\mu_2}{\sqrt{1 - r^2 + \frac{r^2\mu_2^2}{4}}};$$

and writing $r = \frac{1}{2}$, we have

$$R_{13} = \frac{-\frac{\mu_2}{4}}{\sqrt{\frac{3}{4} + \frac{\mu_2^2}{16}}}.$$

In the attempt to determine correlation between the invariable coat-colour of an albino mouse and the colour of its young, we meet with the difficulty that the correlation coefficient takes the indefinite form $\frac{0}{0}$, and cannot therefore be directly evaluated. The only character of an albino connected with its colour which can be evaluated is ancestral purity or impurity, that is the absence or presence of pigmented individuals in its ancestry; and if the albinos be classified into pure and impure it is found that the correlation between the presence of pigment in their ancestry and the amount of whiteness or the depth of colour in their hybrid children is *invariably negative* (cf. Tables VI. and VII.). On any theory of gametic purity the correlations should of course be zero; they are in fact of quite significant magnitude, and considering the probable error of the method adopted they are not unlike. The values obtained are:

Between purity of albino parent and

(*a*) Amount of white coat in hybrid $-0{\cdot}21$

(*b*) Colour of coat . . . $-0{\cdot}16$

the differences being well within the probable error of the method.

It is not suggested that the remarks here offered prove an agreement between the results obtained and those which follow from the law of ancestral heredity. It is contended that the experiment confirms a result, already obtained by Cuénot, that the ancestry of albinos is a factor in determining the characters of the young produced when albino and coloured mice are crossed; it is contended that this result makes it impossible to regard albinism as a character which is transmitted in a state of "gametic purity" by every individual which exhibits it; and therefore since the ancestry of albinos has to be taken into account in predicting their behaviour when crossed, some law of ancestral heredity must be formulated. The present data do not enable us to determine the correlations between parent and offspring with sufficient accuracy to say whether the Galton-Pearson law will fit this case as it fits so many others; but they do enable us to say that the *sign* of these correlations—the first negative coefficients of direct parental inheritance yet published—is in accordance with that law, and that the results obtained do not demonstrably contradict it in other respects.

Offspring of Hybrids.

Hybrids have been paired together or crossed with albinos; a third group of unions between hybrids and extracted albinos may perhaps be treated as belonging to this generation. The results of these three kinds of unions will be considered separately.

Offspring of hybrids paired together.

The unions of hybrids gave altogether 555 young; which were divisible into three groups according to the characters of the eyes and coat: these were

Albino	137
Coloured or piebald with dark eyes	287 or 284*
Coloured or piebald with pink eyes	131 or 134
	555

Dividing these mice according to their mode of walking we find that there are

Waltzing	97
Normal	458
	555

It is clear that the albinos occur in this generation with a frequency indicated by a purely Mendelian theory of albinism regarded as a simple unit character,

* There is unfortunately a doubt concerning the eye-colour of three mice; these are all lilac in colour, and I am practically sure that they had pink eyes. I have never observed a lilac mouse which had dark eyes; the omission of the letter *p* after the lilac mice catalogued in my second paper (*Biometrika*, Vol. II. Part 2, pp. 168, 169) is due to inadvertence in proof reading.

allelomorphic to a single unit character in the waltzing race; that hypothesis leads us to expect that a quarter of the offspring of hybrids paired together will be albinos; and since $\frac{555}{4} = 138{\cdot}75$, the observed result is in excellent accord with it; the difficulty in this, as in all attempts to apply Mendel's hypothesis, is to discover what are the unit characters concerned.

In the preliminary account of the experiment it was evident that the proportion of pink-eyed individuals, with colour in the coat, was also nearly $\frac{1}{4}$; the number now observed, namely at least 131 and possibly 134, is also a fair approximation to a quarter. So far then the results are not inconsistent with the truth of Mr Bateson's suggestion (*Nature*, March 19, 1903) that the character allelomorphic to albinism is pink eye with colour in the coat.

Using Mr Bateson's original notation and denoting albinism by the symbol G, pinkness of eye with some colour in the coat by the symbol G', the generation is at first sight fairly represented by the formula

$$GG + 2GG' + G'G'$$

where GG' represents a heterozygous union between the albino and its allelomorphic element—resulting, for some reason which the Mendelian hypothesis does not provide, in a dark-eyed mouse with a coat-colour variable and generally unlike that of either pure parent.

The difficulties of this hypothesis are, however, great, even when we confine our attention to the frequency of the various kinds of individuals in the group, without considering the ancestral and fraternal correlations. In this as in the preceding generation the variability of the dark-eyed mice (heterozygotes of Bateson's hypothesis) is so great that it is impossible to regard each of them as resulting from the union of similar Mendelian elements; and the relation between pinkness of eye and coat-colour is in the same way inexplicable on the view that there is only one kind of element G'.

The 131 pink-eyed mice are all of colours *a*, *b*, or *f*, that is yellow, fawn-yellow, or lilac; and these must on any Mendelian view represent more than a single kind of gametic element; but if more than one such element be present, the original waltzing mice must have contained more than a single kind of unit G', and we have to consider Mr Bateson's second hypothesis (*Nature*, May 14), which is that in the formula already given the symbol G' must be regarded as denoting any one out of a series of different allelomorphic units, each leading, when mated with its like, to the production of a pink-eyed mouse with some colour in its coat. The hypothesis which involves the smallest number of kinds of elements is that one of the colours *a*, *b*, or *f* is a compound character resulting from the union of the other two: we should then have to consider the three kinds of pink-eyed mice with colour in the coat as of different gametic constitution; if we regard yellow and lilac as "pure," we should have to regard the yellow pink-eyed forms as "pure dominants," say of constitution $G_1'G_1'$, the lilac pink-eyed forms as pure, and of

constitution $G_2'G_2'$, the fawn-yellow pink-eyed forms being heterozygotes of composition $G_1'G_2'$.

This hypothesis fails in several ways to meet the facts; and since the objections to this apply with greater force to any Mendelian hypothesis involving a larger number of allelomorphic units, it is worth while to consider them in some detail. The two kinds of elements G_1' and G_2', which are here assumed to be manifested in the pink-eyed young with coloured coats, must be derived from the waltzing grandparents, for the hypothesis of gametic purity at once excludes any attempt to derive them from the albino ancestors: hence *either* each original waltzer used must have been a heterozygote of composition $G_1'G_2'$, *or* two kinds of pure waltzers $G_1'G_1'$ and $G_2'G_2'$ must have been employed. I am convinced that neither of these consequences of the hypothesis considered is true. Two mice of constitution $G_1'G_2'$ paired together should have given young of character and relative frequency indicated by

$$G_1'G_1' + 2G_1'G_2' + G_2'G_2',$$

so that all three of the colours observed among the pink-eyed offspring of hybrids must have appeared among the offspring of waltzing mice paired together. Now I am aware that lilac and white waltzing mice are bred by dealers, and I am familiar with their characters; I am absolutely sure that no one among the many young bred from the waltzing mice used in these experiments was lilac in colour.

The second hypothesis that two kinds of waltzing mice were employed involves the belief that one of these was lilac; and this is also untrue.

The known history of the waltzing mice used is therefore incompatible with the view that a and f (yellow and lilac) represent pure gametic elements of which b (fawn colour) is the heterozygote. It is similarly impossible to regard a or f as a character of a heterozygote, b and the remaining character being pure; for this hypothesis also would involve the appearance of lilac individuals in a quarter (or in the whole) of the young produced by waltzing mice bred together. The behaviour of the waltzing mice when bred together is alone sufficient to destroy the validity of any hypothesis, involving numerous elements G', which does not depart from Mendel's doctrines.

Mr Bateson has already proposed (*Nature*, April 23, 1903) to regard the waltzing mice used as heterozygous; if he has once refused to accept the statements as to their purity he or others may well do so again. Let us therefore see what happens under these circumstances. The hybrids produced by pairing a heterozygous waltzing form $G_1'G_2'$ with a homozygous albino GG will be of two kinds, GG_1' and GG_2'; it is said that these will be dark-eyed and have some colour in the coat, but at present there is no means of distinguishing between them: all we know on this hypothesis is that the hybrids are of two kinds which are present in equal numbers. These hybrids, being indistinguishable, will be paired at random by any experimenter; so that the offspring of hybrids now considered

will result from unions of three kinds, of which one will probably be twice as frequent as the other; there will be—

$$GG_1' \times GG_1' \text{ giving } GG + 2GG_1' + G_1'G_1',$$
$$2\,(GG_1' \times GG_2') \quad ,, \quad 2\{GG + GG_1' + GG_2' + G_1'G_2'\},$$
$$GG_2' \times GG_2' \quad ,, \quad GG + 2GG_2' + G_2'G_2',$$

the offspring of hybrids being a series of individuals represented in composition and in relative frequency by

$$4GG + 4GG_1' + 4GG_2' + G_1'G_1' + 2G_1'G_2' \times G_2'G_2',$$

where all the zygotes of constitution $G'G'$ have pink eyes and some colour in the coat. The sum of these is a quarter of the whole population, and we have seen that this is compatible with the observed result; but the three colours in the coat should occur with relative frequency 1 : 2 : 1, or among the 131 (possibly 134) individuals actually found there should be if we call a and f G_1 and G_2—

$G_1'G_1'$ ..... 32·25 instead of 19 observed.
$G_1'G_2'$ ..... 64·50 ,, 80 observed.
$G_2'G_2'$ ..... 32·2 ,, 32 or 35 observed.

Taking either of the possible numbers of pink-eyed lilac mice we find that these results are not impossible on the hypothesis put forward, but the odds against results so divergent as these or more so are about 60 or 70 to 1.

The distribution of pink-eyed young might therefore, if not very plausibly, be accounted for by a purely Mendelian hypothesis if that hypothesis did not involve the appearance of lilac individuals in the offspring of pure-bred waltzing mice when paired together. The absence of such individuals might be explained by a departure from the Mendelian statements such as that made by Bateson in several cases, and it might be assumed that the pure waltzing mice have their eye-colour and coat-colour determined by a compound allelomorph $(G_1' + G_2')$ not resolved into its components during gametic-formation in a pure-bred individual. This view is consistent with the behaviour of the pure waltzing strain, but it does not lead to the observed frequencies among the offspring of hybrids. The hybrid zygote will on this view be of constitution $GG_1' + GG_2'$; and two such hybrids when crossed will give in every 16 young

$GG + GG$ ..... once
$GG + GG_1'$ ..... twice
$GG + GG_2'$ ..... twice
$GG_1' + GG_2'$ ..... four times
$GG_1 + GG_2'$ ..... twice
$GG_2' + G_1'G_1'$ ..... twice
$G_1'G_1' + G_2'G_2'$ ..... once
$GG + G_1'G_1'$ ..... once
$GG + G_2'G_2'$ ..... once

and a zygote of elements G_1' only, or G_2' only can never be produced without assuming that remarkable elimination of some elements from every gamete suggested by Mr Bateson (*Proc. Camb. Phil. Soc.* XII.), and so treating the phenomena of colour inheritance in a manner not only foreign to all Mendel's conceptions, but leading to obvious absurdity if applied to any set of characters other than colours.

A further difficulty involved in any of these hypotheses is the correlation between coat-colour and eye-colour. Pink eyes are confined, as has been said, to mice with yellow, fawn-yellow, or lilac coats, and all lilac mice whose eye-colour is certainly known have pink eyes; but dark-eyed mice with yellow or fawn-coloured coats occur in this and in the preceding generation, and their presence is difficult to explain on either of the hypotheses examined.

The difficulties we find when we try to describe the various classes of individuals found among the offspring of hybrids are increased when we consider the great variability both in the colour of the coloured patches and in their extent among the individuals of any class, the amount of which may be gathered from Tables I.—VII.

The correlations between the various characters exhibited by the offspring of hybrids and the characters of their ancestors are quite inexplicable on any hypothesis involving the purity of the albinos. The correlation between individuals of this generation and the purity or impurity of their albino grandparents—that is the measure of the effect of ancestral pigmentation transmitted through the albino parent—is evident from Tables XX.—XXIII. These correlations are all negative,—the sign, taking the scales in the directions given in the tables, showing that the amount of whiteness is less among the individuals descended from pure-bred albino grandparents, the coat-colour itself being darker; so that the relation between the albinos used in the original cross and the offspring of hybrids has the same sign as that between the albinos and the hybrids themselves; the correlation, however, is apparently larger, instead of being smaller as might have been expected, the value of r for both colour and whiteness being about $-0{\cdot}3$ (Tables XX., XXI.). A correlation so large as this is far beyond the probable error of the determination, and it is of course absolutely fatal to any theory which involves the "gametic purity" of the albinos.

The only character of the pure-bred waltzing grandparents which varies enough to permit of its correlation with the offspring of hybrids being determined, is the extent of the coloured patches. Dividing the waltzing grandparents into α and β as already explained, the correlations are fairly uniform, the coefficient for colour and for amount of whiteness being about $0{\cdot}3$ (see Tables XXIV. and XXV.), as in the previous generation; the sign of the correlation between colour in the hybrids and amount of whiteness in their waltzing parent (which of course depends upon the order in which the colours are arranged upon the arbitrary scale) is *negative*, showing that the waltzing mice with a greater amount of white

produce hybrids with darker coat-colour; between waltzing grandparents and the offspring of hybrids the sign is, however, positive for the same arrangement of colours, so that while the intermediate hybrid offspring of waltzing mice with a large amount of white coat are darker than the average of hybrids, the young produced by such offspring paired together are on the whole lighter than the average.

It must be remembered that this relation between the amount of white coat in one generation is a relation of cross inheritance, depending on (though not always simply proportional to*) the product of the coefficients of direct inheritance and of organic correlation. We have seen that the coefficient of organic correlation between amount of whiteness and coat-colour is negative in the hybrids themselves; in their offspring it has changed its sign and become positive, so that we see the way in which the sign of the coefficient of cross grandparental inheritance has become changed. We see then that both the colour of the coat and the amount of white fur in the offspring of hybrids are correlated about as strongly with characters of one set of grandparents as with those of the other; neither the waltzing grandparents nor the albinos are dominant in the sense that they alone determine the characters of the generation, neither are recessive in the sense that they take no part in such determination. It may be urged that the correlations just determined are without value because the offspring of hybrids are obviously divisible into two groups, albinos and others—and perhaps into three. There is some indication that the correlations between the albinos and their waltzing grandparents is negative, the grandchildren of the whiter waltzers containing a distinctly smaller percentage of albinos than the grandchildren of waltzers with a larger amount of coat-colour: the numbers descended from the various waltzing grandparents are shown in Tables XXIV. and XXV., and it will be seen that the deviation from Mendelian expectation is only great enough to be suggestive among the 112 offspring of hybrids whose waltzing grandparents were both of type α. The number of albinos which should be found in this group is 28, and the probable error of the expectation is $0{\cdot}6745\sqrt{\frac{1}{4}\times\frac{3}{4}\times 112} = 3{\cdot}09$; the number of albinos actually found is 18, showing a deviation from the most probable result equal to more than three times the probable error, the chance of a deviation so great as this being about 0·0015, or the odds against it about 370 to 1.

If, however, we exclude the albinos, and calculate the correlation between the coat-colour or the amount of white fur in the remaining individuals, and the whiteness of their waltzing grandparents, we obtain values very closely similar to those already given, so that in this case at least the apparent dimorphism of the young has no effect upon the grandparental correlations.

The parental correlations are considerable, and curiously different for different characters. The correlation between the amount of white coat in hybrids and in their offspring is about 0·7 for both father and mother, showing that the whiter

* Cf. Pearson, "On the Laws of Inheritance in Man," *Biometrika*, Vol. II. Part IV. pp. 384—5.

parents produce a markedly greater proportion of young in which whiteness predominates than are produced by the darker parents; they also produce a larger proportion of pure albinos.

In coat-colour the correlation between parents and offspring is much less, being about 0·2 for both parents. The cross-inheritance, as measured by the correlation between coat-colour of father or of mother and whiteness of young, is again positive (in accordance with the positive organic correlation between these two characters in the young), and its value is about 0·2.

These various correlations cannot be adequately discussed without attempting a study of the effects due to the selection of parents during the experiment, too difficult for any but a trained mathematician. The hybrids were to a large extent paired together so that the members of a pair were of similar colours, the correlation between male and female members of a pair (weighted with the number of young produced) being about 0·4, so that the coefficient of assortative mating has changed from + 1 in the ancestors of the pure-bred mice, through − 1 in the unions which produced the hybrids, to about 0·4 in unions resulting in the generation now described. The amount of whiteness in the coat of the hybrids was also selected, the correlation between male and female in the unions made being over 0·9; this difference in the degree of assortative mating may possibly account for the difference between the parental correlations in the two cases.

We see, however, that both the colour and the amount of whiteness in the coat are variable characters, depending to a sensibly equal extent on characters transmitted by each of the pure-bred parents of the hybrids, so that there is no question of dominance on the part of one or other parent, and no possibility of regarding the characters of the generation examined as depending solely on the individuals paired in the original cross union, apart from their ancestry. In other words, we see that here, as in the hybrid generation, the phenomena observed must be formulated in some way or other in terms of the whole ancestry, and the idea of "gametic purity" is therefore excluded.

Waltzing occurs in only 97 out of the 555 individuals resulting from the union of hybrids. When we compare this with the number of pink-eyed individuals (131—134) or of albinos (137) we see that the proportion of waltzing individuals cannot be regarded as a possible quarter. The probable error of the expectation that a quarter of the individuals will waltz is, on the Mendelian hypothesis, $0{\cdot}6745\sqrt{\frac{1}{4}\times\frac{3}{4}\times 555} = 6{\cdot}88$ only, and the observed deviation is $138{\cdot}75 - 97 = 41{\cdot}75$, the odds against so great a deviation being rather more than 50,000 to 1. As the result here obtained differs from Mendelian expectation in the same direction as that already obtained by von Guaita* and to an extent consistent with the agreement of both, the evidence that the waltzing character does not segregate in Mendelian proportions is very strong.

* "Versuche mit Kreuzungen von verschiedenen Rassen der Hausmaus," *Ber. d. naturforsch. Gesellsch. Freiburg*, Bd. x. 1898, Bd. xi. 1900.

Waltzing is completely recessive to normal walking, disappearing entirely in the hybrids resulting from the cross; its correlations with other characters, parental, grandparental or fraternal, are always small and generally insensible. Except for the fact that the proportion of waltzers among the offspring of hybrids is so widely divergent from that indicated by the theory, waltzing is thus more nearly Mendelian in its behaviour than any of the characters examined. The distribution of waltzing individuals among the various families (see Table A) is, however, less regular than it should be, and in this way a sensible fraternal correlation is established (Table XIX.) which is not in accord with Mendelian theory.

HYBRIDS CROSSED WITH ALBINOS.

Unions between hybrids and albinos produced 746 young, of which 368 were albino, the rest being piebald or self-coloured. The gap between albinos and coloured individuals was wider than among the offspring of hybrids paired together, since the Group 1, including individuals in which the pigmented area is smallest, was entirely absent, and only eight individuals fell into the next group 2, the majority of individuals which were not albino being of degree 5 or 6. The coloured individuals had dark eyes, and none showed the waltzing habit. The range of coat-colours was less than among the offspring of hybrids paired together, since lilac did not occur. The relative frequency with which the different colours occurred was strikingly different from that seen among the offspring of hybrids, as will be gathered from the following table:

Percentage frequency of the Colours in (a) Coloured Offspring of Hybrids paired together, (b) Coloured Offspring of Hybrids and Albinos.

Colour	Hybrid × Hybrid	Hybrid × Albino
a	7·66 %	6·08 %
b	22·97 %	3·17 %
c	36·36 %	44·18 %
d	4·55 %	14·55 %
e	18·66 %	29·10 %
f	8·37 %	0·00 %
g	1·44 %	2·90 %
	100·01	99·99

It will be seen that the coloured offspring of hybrids and albinos, which have more white in their ancestry than the offspring of hybrids alone, are characterised *first* by having considerably less white fur, and *secondly* by the greater frequency of wild-colour and black, as compared with yellow and fawn-colour—that is, by the presence of black pigment in larger quantity. This increase of blackness with increase in the number of albino ancestors is very remarkable, and no explanation suggests itself at present; it is however in accord with the negative

correlation already mentioned between purity of ancestry among the albinos employed in making the original cross and the darkness of the hybrids.

The albinos of the group seem more obviously distinct from the rest than do the albinos among the offspring of hybrids, and the ancestral correlations are materially affected by their retention or omission.

Retaining the albinos the correlation between whiteness of coat and purity of the albino grandparent is about $-0{\cdot}15$ (cf. Table XLV.); between whiteness of coat in coloured individuals alone and purity of albino grandparent the coefficient rises to about $-0{\cdot}25$. We have here a further proof of the extent to which the ancestry of the "recessive" albino affects the coloured descendants of the cross. The correlations with the albino *parent* are too small to be worth noticing until the probable error of the method employed for their determination can be given; the data are given in Tables XLIX. and L., where it will be seen that nearly all the albinos used as parents were pure-bred, so that the determination of correlations is difficult.

The hybrid parent appears to be more closely correlated with the young, both in colour and in amount of whiteness, than the albino, the correlation in both characters being about $0{\cdot}2$, the value found among offspring of hybrids paired together.

Hybrids crossed with extracted Albinos.

The object of this experiment was to determine if the percentage of albinos produced by such crosses would be different from that produced by crosses between hybrids and pure albinos. The number of albinos obtained—59 out of 120—is of course an excellent experimental half; but the following considerations should be carefully weighed, (i) the albinos used were not all produced by pairing two hybrids; in many cases (see Table D) they were produced by crossing hybrid and albino, (ii) the hybrids used were in some cases (G_{12}, G_{14}, G_{15}, G_{16}, G_{17}, G_{19}, G_{20}) the offspring of waltzers and extracted whites, (iii) the character the percentage of which one is trying to diminish in the offspring is identical with that of one of the parents: in offspring of extracted hybrids one was able to diminish the number of albinos by diminishing the amount of albino ancestry of the extracted hybrids; but in this experiment, while one does diminish the albino ancestry, one of the *parents themselves* is always albino; so that the effect of the diminution of albino ancestry is rather swamped.

If separate tables are made for, (i) hybrids crossed with albinos produced by the union of hybrids, (ii) hybrids crossed with albinos produced by hybrids paired with albinos, and (iii) hybrids from extracted white crossed with extracted albinos, the numbers become so small as to be useless to base arguments upon.

The occurrence of pink eyes in seven of the young, or five per cent. of the number produced, distributed among five families (more than one-fifth of the whole number), is clear evidence that these extracted albinos have a power of hereditary

transmission, dependent upon their ancestry, different from that of albinos whose ancestry contains no pink-eyed waltzers.

GRANDCHILDREN OF HYBRIDS.

The children of hybrids consisted roughly of a quarter albinos, a half dark-eyed forms with coloured coats and a quarter pink-eyed mice with colour in the coat. The extracted albinos are unquestionably the recessives: the dark-eyed mice with coloured coats I have termed the extracted hybrids; partly because they occur in the proportion of 50 % and partly because they are asserted by Bateson and Castle to be what I have called them. The pink-eyed forms with colour in the coat which occur in the Mendelian proportion of a quarter I have called extracted dominants for similar reasons: I feel quite confident that Mendel himself would have sanctioned this classification, since the proportions agree closely with those which he obtained with his peas.

The grandchildren of hybrids will be treated of under the following headings: (i) offspring of extracted recessives $RR \times RR$, (ii) offspring of extracted hybrids $DR \times DR$, (iii) offspring of extracted dominants $DD \times DD$.

(i) *Offspring of Extracted Recessives.*

As could have been predicted with no fear of contradiction by those who are familiar with fancy mice the extracted albinos when paired together produce only albinos. For some reason I have been unable to breed from waltzing individuals of this category, and therefore the number of young which waltz is small, but it is far below that which would appear on Mendelian expectation, which is 1 in 12. For the albinos may be as regards waltzing either dominant or hybrid, i.e. the kinds of pairs of albinos that can be made are three:

(i) $DD \times DD$.
(ii) $DD \times DR$.
(iii) $DR \times DR$.

Of these pairs, (i) and (ii) should produce no waltzers, and (iii) should produce 1 in 4. The whole number of young obtained was 94, and of these only *two* waltzed!

(ii) *Offspring of Extracted Hybrids.*

I think that the most conclusive results which I have obtained are given by the results of pairing extracted hybrids: as the reader is aware, hybrids were paired with hybrids ($DR \times DR$) and also crossed with albinos ($DR \times RR$), the former giving offspring which may plausibly be regarded as consisting of $\frac{1}{4} DD + \frac{1}{2} DR + \frac{1}{4} RR$, the latter $\frac{1}{2} RR + \frac{1}{2} DR$. So that, when pairing the children of hybrids to produce the generation which we are now considering, two kinds of extracted hybrids were available, (i) the result of $DR \times DR$ (which I will denote $H \times H$ = hybrid × hybrid) and (ii) the result of $DR \times RR$ (which I will call

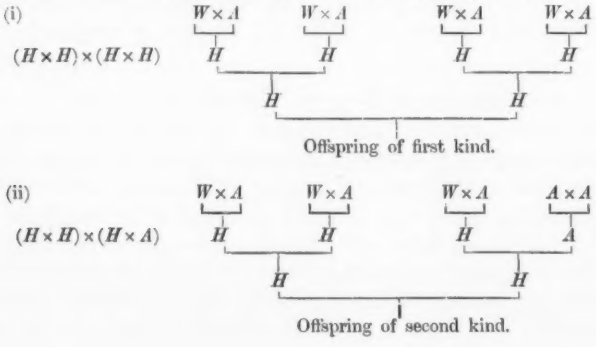
$H \times A$). With these two kinds of extracted hybrids three kinds of crosses can be made:

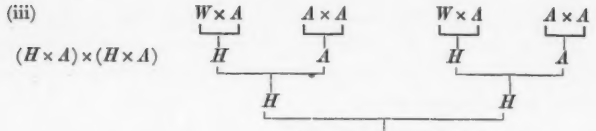
1. $(H \times H) \times (H \times H)$.
2. $(H \times H) \times (H \times A)$.
3. $(H \times A) \times (H \times A)$.

Now in Mendelian language these kinds of pairs are all $DR \times DR$, for the DR produced by $DR \times RR$ is not different from the DR produced by $DR \times DR$. And the similarity of these formulae is characteristic of the Mendelian conception of the reproductive organs of the hybrids.

For it is well known that according to this view the hybrid contains equal numbers of germ-cells which produce the dominant character, and of those which produce the recessive; and this is said to be true of the hybrids however far the individual is removed from the original cross, whether it is the result of the cross (i.e. the hybrid) or the great-great-grandchild of this. This is the ground on which the doctrine of the purity of the germ-cells and the law of ancestral heredity flatly contradict one another; the former asserting that $DR \times DR$ will produce 25 °/₀ DD, 50 °/₀ DR and 25 °/₀ RR for a very great if not an infinite number of generations; the latter maintaining that the further the individual hybrid under consideration happens to be removed from the cross the less albinos will it produce: and that two hybrids whose mothers were albinos will produce more albinos than would two hybrids who have no albinos in their pedigree later than their great-great-great-grandmother. This seems to me to afford a case in which experiment could provide a decisive answer. I have made an experiment of this kind: that is, I have tried to see if by pairing together hybrids with different amounts of albino ancestry I could obtain different percentages of albino individuals among the offspring. I have not had time to do this by producing successive generations of hybrids and counting the number of albinos in each: but I have done it by observing the difference between the results of making crosses of form $(H \times H) \times (H \times H)$, $(H \times H) \times (H \times A)$ and $(H \times A) \times (H \times A)$.

The ancestry of these kinds of pairs is perhaps brought more vividly before the reader's mind by considering the following pedigrees:





Offspring of third kind.

It is evident that the amount of albino ancestry is largest in (iii), smaller in (ii), and smaller still in (i). The doctrine of the gametic purity asserts that a series of individuals having any of the pedigrees above represented should contain 25 per cent. of albinos and 75 per cent. of coloured mice. The law of ancestral inheritance proclaims that the percentage of albinos will be greater in (ii) than in (i) and greater in (iii) than in (ii): which is exactly what we find to be the case.

The number of albinos produced by each kind of pair is seen at a glance in the following table.

Origin of the hybrids mated	Number of young	Number of albinos	Albinos per cent.
$(H \times H) \times (H \times H)$	93	10	10·75
$(H \times H) \times (H \times A)$	107	20	18·69
$(H \times A) \times (H \times A)$	121	30	24·79

The amount of whiteness in the various coloured individuals resulting from these unions, and the colour of their coats, may be gathered from Table E on p. 35. An interesting feature is the reappearance of lilac in the grandchildren of unions between hybrids and albinos, though this colour has been seen to be absent from the immediate progeny of such unions.

Extracted Hybrids crossed with extracted Albinos.

In the unions just described, hybrids of various kinds were paired together. By pairing such hybrids with extracted albinos we should, on the Mendelian view, produce equal numbers of albinos and of dark-eyed hybrids. Out of 91 young actually produced, 36 were albino; this is not an impossible approximation to a Mendelian half, but it is so bad as to afford a strong suggestion that the ancestry of the albinos used has affected the result, and this suggestion is confirmed by the fact that six of the young produced have pink eyes, a phenomenon never observed when albinos of ancestry which does not contain waltzers are crossed with these hybrids. See Table F, p. 36.

Offspring of Extracted Dominants.

(*a*) *Extracted Dominants paired together.*

Pink-eyed forms with colour in the coat have been paired together, but the results so far obtained are too few to be of great value. The whole number of young, obtained from seven unions, was 32 (see Table H, p. 37). The young were all pink-eyed, but one was completely albino.

It has already been pointed out that there is more than one kind of pink-eyed mouse with colour in the coat, and the difficulties which arise, when we attempt to derive these various kinds of mouse from the pure-breeding waltzer by any Mendelian process, have been indicated.

So far as the observations go, they suggest (1) that a proportion of albinos, not yet determined, results from these unions; (2) that lilac mice, paired together, breed true to coat-colour; (3) that dark-eyed mice are not produced in any large proportion; (4) that the amount of whiteness in the coloured young is highly variable.

(*b*) *Extracted Dominants paired with Albinos.*

"Extracted dominants" have been paired with albinos in order to find out whether such unions behave according to Mendelian expectation. If the so-called extracted dominants were really of the constitution suggested by the name given to them, they should of course, when paired with albinos, produce a series of hybrids like those resulting from the original cross, albinos being absent. This was not found to occur. Out of 98 young produced, 12 were albinos—practically the same percentage as that given by one series of dark-eyed hybrids paired together—while one pink-eyed individual occurred.

Most of the albinos used in making these crosses were "extracted" from the offspring of hybrids paired with albinos, as will be seen from the pedigrees given in Table G, p. 36; of the 98 young produced, 63 were from unions between such "extracted" albinos and the extracted dominants; the seven unions between extracted dominants and albinos which did not include pink-eyed fawn-and-white waltzers among their immediate ancestors produced 35 young, of which no less than 10, distributed among four of the unions, were albino; of the three unions of this kind which produced no albinos, one was between a pure-bred albino and the same "dominant" individual which produced albinos when mated with another pure-bred albino; there remains only two unions (Nos. 2*H* 79 and 2*H* 80) between pure albinos and "extracted dominants," which did not produce albino young; as these two unions produced only 12 mice, their evidence is not conclusive.

The experiments seem to show (1) that pink-eyed mice with colour in the coat are not always, if ever, "pure dominants," since they do at least generally produce some albinos when paired with albino mice of appropriate race; (2) that the "extracted recessive" albino mice behave differently from albinos of other ancestry.

The second result, together with other evidence already given, disproves the "pure recessive" nature of extracted albinos, the first makes it extremely doubtful whether "pure dominants" exist.

Conclusions.

The experiments described appear to justify the following statements:

1. When the race of waltzing mice used is crossed with albino mice which do not waltz, the waltzing habit disappears in the resulting young, so that waltzing is

completely recessive in Mendel's sense; the eye-colour of the hybrids is always dark; the coat-colour is variable, generally a mixture of wild-grey and white, the character of the coat being distinctly correlated with characters transmitted both by the albino and by the coloured parent. There is thus no proper dominance in Mendel's sense, so far as eye-colour and coat-colour are concerned, the hybrids differing always in eye-colour and generally in coat-colour from *both* their parents.

2. When the hybrids, produced from the cross described, are paired together, the resultant young exhibit a segregation into three groups, so far as eye-colour and coat-colour are concerned, into two so far as regards the waltzing habit. The phenomenon of segregation is closely similar to that described by Mendel, and in the colour, whether of eyes or of fur, the proportions are closely identical with those observed by him, a quarter of the young resembling their albino grandparent, half representing their hybrid parents, and a quarter resembling their waltzing grandparents in so far that they have pink eyes and some coloured fur, but differing from any of their immediate ancestors in the range of coat-colours exhibited. The proportion of individuals which exhibit the waltzing habit is less than one-fifth of the whole number of young, and is not a Mendelian proportion.

3. When hybrids are paired with albinos, half the young produced resemble their albino parent, half resemble their hybrid parent; this result is in accord with Mendel's theory.

4. The correlations between coat-colour in the hybrids and descendants on the one hand, and the ancestry of their albino grandparents on the other, are not consistent with the view that albinism is a recessive Mendelian character, transmitted "gametically pure" by any homozygote which possesses it.

5. The behaviour of "extracted albinos," "extracted hybrids," and "extracted dominants" is not consistent with any theory of gametic purity yet propounded. There is no evidence that any individuals which could be properly described as "pure dominants" or "pure recessives" exist among the whole series produced.

6. The effect of varying ancestry is so great, in every case in which it has been examined, as to show that the phenomena observed cannot be adequately described except in terms of the ancestry of all the individuals used as parents.

7. There is no evidence to show whether the ancestral correlations observed are consistent or not with the consequences which would follow from the application to this particular case of the Galton-Pearson law of inheritance; the only consequence of that law which can at present be deduced, namely, the negative correlation between one parent and the offspring of the first cross, is in agreement with observation.

8. In this, as in many other cases, the characters of the hybrids produced by crossing two races which have been long distinct strongly suggest reversion to ancestral characters.

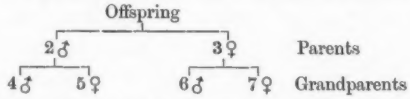
I cannot bring these observations to a close without expressing my gratitude to those to whom it is due. Professor Weldon has helped to bridge over a gap

due to my absence from Oxford for two months, and has given me assistance in tabulating the correlations. I owe thanks to Professor Pearson for giving aid in the calculation of the correlation coefficients, and in the criticism of some of the conclusions drawn from them. I am also very grateful to my friend Mr E. H. J. Schuster for much arithmetical help very generously given. Last, and almost, therefore, not least, to Mr Frank Sherlock I owe a debt of gratitude which is difficult to express. Mice are living things, and Mr Sherlock is almost entirely responsible for supplying them with their daily needs for more than a year with an industry and patience which is beyond all praise. The sanitary condition of a room containing some fifteen hundred mice is sufficient testimony to his conscientiousness and competence: he is in a way more directly responsible for the carrying out of the experiment than anyone else. For these things he merits my lasting gratitude, which is tendered to him in full measure.

APPENDIX I.

List of all Mice bred during the Experiment, with the Characters of their immediate Ancestors.

Explanation of the Tables:—Every line of the table contains a single family; the columns on the left, numbered 2—7, describe the characters of the ancestors. The notation used is nearly that of Mr Galton (see *Biometrika*, Vol. II. p. 237), and is shown by the following pedigree, in which every even number denotes a male, every odd number a female ancestor:



Every individual is described by a series of symbols denoting the amount of white fur in its coat, the nature of the colour in its coat, its eye-colour, and its power of waltzing. The following symbols are used:

A = Albino; in the columns recording ancestry, A alone means a pure-bred albino, while $A\times$ denotes a cross-bred albino (used only in recording ancestors).

a = Yellow.
b = Fawn colour.
c = Wild-grey.
d = Dark wild-grey.
e = Black.
f = Lilac.
g = Chocolate.

α is used only of pure-bred waltzers with much white in the coat.

β is used only of pure-bred waltzers with less white in the coat than is signified by α.

W = A pure-bred waltzer, fawn and white, with pink eyes.

w = A cross-bred with waltzing habit.

p = With pink eyes. (*Note* that since all albinos and all the pure-bred waltzers used had pink eyes this symbol is omitted in describing pure-bred ancestors. When it is omitted in other cases, the individual referred to had dark eyes.)

The figures, Nos. 1—6, denote amounts of white fur in the coat, as explained on p. 1.

TABLE A.—*Results of crossing Waltzing Mice with Albinos.*

Catalogue Number	Parentage		Offspring	Number in Litter
	♂	♀		
1	*A* ×	*Wa*	1*c*, 1*c*, 2*c*, 2*d*, 3*c*, 3*c*	6
6	*A* ×	*Wβ*	2*c*	1
7	*A*	*Wa*	2*c*, 2*c*, 2*d*, 3*c*, 3*d*, 5*c*, 5*c*	7
8	*A*	*Wβ*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
9	*A* ×	*Wa*	2*c*, 2*c*, 2*c*, 2*c*, 2*c*, 2*d*	6
12	*A*	*Wa*	5*a*, 5*a*, 5*c*, 5*c*, 5*c*	5
13	*A*	*Wa*	3*c*, 3*c*, 5*c*, 5*d*	4
16	*A* ×	*Wβ*	3*c*, 3*c*, 3*c*, 4*c*, 4*c*	5
20	*A*	*Wa*	2*c*, 2*d*, 2*d*, 3*d*, 5*d*, 5*d*	6
27	*Wa*	*A*	2*e*	1
33	*Wa*	*A*	5*c*	1
45	*A*	*Wa*	2*d*, 2*d*, 2*d*, 2*d*	4
53	*Wβ*	*A*	4*c*	1
59	*Wa*	*A*	3*d*, 6*d*	2
61	*A*	*W*	2*c*, 3*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*	7
62	*A*	*Wβ*	2*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*	6
66	*W*	*A*	5*c*	1
72	*W*	*A*	5*c*	1
73	*A*	*W*	3*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*	6
74	*A*	*W*	3*c*, 4*c*, 4*c*, 4*c*, 4*c*, 4*c*, 4*c*	7
75	*A*	*W*	1*c*, 2*c*, 2*c*, 3*c*, 4*c*	5
77	*A*	*Wβ*	4*c*, 4*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	8
78	*A*	*Wβ*	2*e*, 3*e*, 5*d*, 5*e*	4
82	*Wβ*	*A*	3*c*, 5*c*	2
84	*A*	*Wβ*	5*a*, 5*a*, 5*a*, 5*a*, 5*a*, 5*c*, 5*c*, 5*c*, 5*c*	9
85	*A* ×	*Wa*	3*e*, 3*e*, 4*d*, 5*d*, 5*d*, 6*d*, 5*e*	7
86	*A*	*W*	3*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7
90	*A*	*Wa*	5*c*, 5*c*	2
91	*Wa*	*A*	6*c*	1
94	*Wa*	*A*	3*c*, 3*c*	2
100	*A*	*Wβ*	2*c*, 3*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	9
101	*A*	*Wβ*	2*c*, 3*c*, 3*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	9
102	*A*	*Wβ*	3*c*, 4*c*, 4*c*, 4*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*	9
103	*A*	*W*	2*c*, 3*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*	8
104	*A*	*W*	1*c*, 2*c*, 2*c*, 3*c*, 3*c*, 4*c*, 5*c*	7
107	*Wa*	*A* ×	3*e*, 5*c*	2
108	*Wa*	*A* ×	5*c*	1
110	*Wa*	*A*	3*c*, 3*c*	2
111	*A*	*Wa*	5*a*, 5*a*, 5*a*, 5*a*, 5*a*, 5*d*, 5*d*, 5*d*, 5*d*, 5*d*	10
112	*A* ×	*Wβ*	2*c*, 2*c*, 2*c*, 3*c*, 5*c*	5
113	*A* ×	*W*	3*c*, 3*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	8
114	*A* ×	*W*	2*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*	6
117	*A* ×	*W*	3*c*, 3*c*, 3*c*, 4*c*, 4*c*, 4*c*, 4*c*	7
118	*A* ×	*Wa*	4*c*, 4*c*, 5*c*, 5*c*, 5*c*	5
120	*A* ×	*Wa*	2*c*, 2*c*, 3*c*, 3*c*, 3*c*, 4*c*, 4*c*	7
142	*A* ×	*Wa*	2*c*, 2*c*, 2*c*, 3*c*, 5*c*, 5*c*	6
144	*A*	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
147	*A* ×	*Wβ*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
159	*A*	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
160	*A*	*Wβ*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*	5
161	*A*	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
162	*A*	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
163	*A*	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*	5
165	*A*	*W*	4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7
166	*A*	*W*	3*c*, 4*c*, 5*c*, 5*c*, 5*c*	5
167	*A*	*W*	4*c*, 4*c*, 5*c*, 5*c*, 5*c*	5
169	*Wβ*	*A*	6*c*, 4*c*	2
170	*A* ×	*Wa*	3*c*, 3*c*, 4*c*, 5*c*, 5*c*	5
171	*A* ×	*Wa*	3*c*, 3*c*, 3*c*, 3*c*, 5*c*	5
172	*A* ×	*Wa*	4*c*, 4*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	8
175	*A* ×	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7
176	*A* ×	*W*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7
179	*A* ×	*Wa*	3*c*, 3*c*, 3*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*	8
184	*A* ×	*Wβ*	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	10
			Total	340

TABLE B.—*Offspring of Hybrids paired together.*

Catalogue Number	Parentage						Offspring	Number per Litter
	3	*2*	*7*	*6*	*5*	*4*		
H 7a	1*c*	2*c*	*A* ×	*Wa*	*A* ×	*Wa*	2*fpw*, 2*ap*, 2*b*	3
H 7β	„	„	„	„	„	„	2*bp*, 2*c*, 2*d*, 2*d*, 3*c*	5
H 8	2*c*	2*c*	*A* ×	*Wa*	*A*	*Wa*	3*bp*, 4*ap*, 4*ap*, 3*c*	4
H 9	2*c*	3*c*	*A* ×	*Wa*	*A*	*Wa*	3*ap*, 3*b*, 3*b*, 3*cw*, 3*fp*	5
H 18	3*c*	2*c*	*A* ×	*Wa*	*A*	*Wa*	3*bpw*, 3*bp*, 3*c*, *A*, *A*, *A*, *A*	7
H 19a	3*c*	3*c*	*A* ×	*Wa*	*A*	*Wa*	2*c*, 2*c*, 2*cw*, 4*ew*, *A*, *A*	6
H 19β	„	„	„	„	„	„	2*e*, 2*bp*	2
H 20	2*c*	2*c*	*A* ×	*Wβ*	*A* ×	*Wa*	2*cw*, 2*e*, 2*fp*, 3*cw*, *Aw*, *A*	6
H 29a	2*c*	3*c*	*A* ×	*Wa*	*A* ×	*Wβ*	2*c*, 2*e*, 3*d*, 4*e*, *A*	5
H 29β	„	„	„	„	„	„	2*cw*, 3*fp*, *A*	3
H 30	2*c*	5*c*	*A* ×	*Wa*	*A*	*Wa*	2*dw*, 2*fp*, 2*fp*, 3*bp*, 3*d*, 5*cw*	6
H 42a	5*a*	3*c*	*A*	*Wa*	*A* ×	*Wa*	4*ap*, 3*c*, 3*c*, *Aw*	4
H 42β	„	„	„	„	„	„	1*cw*, 3*apw*, 4*a*, 6*a*, *A*	5
H 61	5*c*	5*c*	*A*	*W*	*A*	*Wa*	5*c*, *A*	2
H 70	3*d*	2*d*	*Wa*	*A*	*A*	*Wa*	2*c*, 3*a*, 3*e*	3
H 71	4*c*	2*d*	*A*	*W*	*A*	*Wa*	3*c*, *A*	2
H 72	2*d*	2*c*	*A*	*Wa*	*A*	*W*	3*c*	1
H 73	5*c*	5*c*	*A*	*W*	*Wa*	*A*	3*c*, 5*c*, 5*d*	3
H 74	5*c*	5*c*	*A*	*W*	*A*	*Wβ*	3*ap*	1
H 76	2*c*	4*c*	*A*	*W*	*A*	*Wβ*	3*e*, *A*, *A*	3
H 77	3*c*	4*c*	*A*	*W*	*A*	*Wβ*	4*e*	1
**H 95*	(?) *c*	(?) *c*	*A*	*Wβ*	*A*	*Wβ*	2*e*, 3*e*, 6*bcp*, 6*bcp*, *Aw*, *A*	6
H 96a	5*a*	5*a*	*A*	*Wa*	*A*	*Wβ*	5*b*, 5*c*, *A*, *A*	4
H 96β	„	„	„	„	„	„	5*a*, 5*a*, 5*a*, *A*	4
H 97a	5*a*	5*a*	*A*	*Wa*	*A*	*Wa*	5*b*	1
H 97β	„	„	„	„	„	„	3*bp*, 5*bp*, 5*c*, *A*	4
H 98a	5*c*	5*d*	*A*	*Wa*	*A*	*Wa*	4*e*, 4*e*, 6*bp*, 6*c*, 6*c*, 5*c*, *A*	7
H 98β	„	„	„	„	„	„	5*bp*, 5*c*, 5*c*, 6*fp*, *A*, *A*	6
H 99	5*c*	5*c*	*Wβ*	*A*	*A*	*Wβ*	5*c*, 5*d*, 6*bp*, *A*	4
H 100a	3*c*	3*c*	*Wβ*	*A*	*A*	*Wa*	2*bp*, 2*c*, 2*d*, 3*d*, 4*bp*, *Aw*	6
H 100β	„	„	„	„	„	„	3*c*, 4*c*, 4*e*, *A*, *A*, *A*	6
H 101a	6*d*	5*c*	*A* ×	*Wa*	*A*	*Wβ*	3*e*, 5*e*, 6*e*, 6*fp*, *Aw*	5
H 101β	„	„	„	„	„	„	6*bfp*, 6*c*, 6*e*, *A*, *A*, *A*	6
H 102a	5*d*	5*d*	*A* ×	*Wa*	*A*	*Wa*	1*cw*, 5*bpw*, 5*dw*, 5*bc*, 6*e*, *Aw*	6
H 102β	„	„	„	„	„	„	2*bp*, 3*c*, 6*fp*, 6*fpw*, 6*fpw*	5
H 103a	4*d*	4*c*	*A* ×	*Wa*	*A* ×	*Wa*	3*bw*, 3*bpw*, 4*bc*	3
H 103β	„	„	„	„	„	„	3*b*, 3*bc*, 3*e*, 4*bc*, 4*bp*, 4*bp*	6
H 104a	2*c*	3*c*	*A* ×	*Wa*	*A* ×	*Wa*	3*bp*, 2*e*, 2*cw*, *A*, *A*	5
H 104β	„	„	„	„	„	„	1*cw*, 2*cw*, 3*bc*, *A*	4
H 105a	3*c*	4*c*	*A* ×	*Wa*	*A* ×	*Wa*	2*bp*, 2*c*, 3*bp*, 3*fp*	4
H 105β	„	„	„	„	„	„	3*bp*, 3*c*, 4*bp*, 4*bp*, 4*e*, *Aw*	6
H 106a	3*c*	3*c*	*A* ×	*Wa*	*A* ×	*Wa*	3*cw*	1
H 106β	„	„	„	„	„	„	2*fp*, 3*c*, 3*dw*, *A*	4
H 107	4*c*	3*c*	*A* ×	*Wβ*	*Wa*	*A*	1*bp*, 4*bc*	2
H 108	2*c*	3*c*	*A* ×	*Wa*	*A*	*Wβ*	2*e*, 3*bp*, *A*, *A*, *A*	5
H 109a	3*c*	3*c*	*A*	*Wβ*	*A*	*Wβ*	2*bpw*, 2*c*, 3*bpw*, *A*, *A*, *Aw*	6
H 109β	„	„	„	„	„	„	2*e*, 3*c*, *A*, *A*, *A*	5
H 110	5*c*	5*c*	*A*	*Wβ*	*A*	*Wβ*	6*apw*, 6*b*	2
H 111a	5*c*	5*c*	*A*	*W*	*A*	*Wβ*	4*bp*, 4*bpw*, 5*bpw*, 5*c*, 6*ap*, 6*c*	6
H 111β	„	„	„	„	„	„	3*bp*, 3*e*, 5*cw*, 6*e*, 5*g*†, *A*, *A*	7
H 112a	3*c*	1*c*	*A*	*W*	*A*	*W*	1*cw*, 2*cw*, 2*fp*, *A*, *A*, *A*	6
H 112β	„	„	„	„	„	„	2*c*, 2*bfp*, *A*	3
H 113a	4*c*	3*c*	*A*	*Wβ*	*A*	*Wa*	3*bcpw*, 3*cw*, 3*e*, 4*c*, *A*	5
H 113β	„	„	„	„	„	„	1*c*, 2*c*, 3*e*, 4*c*, *A*, *A*	6
H 114	2*c*	2*c*	*A* ×	*Wβ*	*A*	*Wβ*	1*bp*, 1*e*, 2*bp*, 2*bp*, 2*c*, *Aw*	6
H 115a	4*c*	3*c*	*A*	*W*	*Wβ*	*A*	1*c*, 2*ew*, 3*c*, 4*e*, *A*	5
H 115β	„	„	„	„	„	„	3*e*, 4*e*, 4*c*, *A*	4
H 116a	4*c*	4*c*	*A*	*W*	*A*	*Wβ*	3*bp*, 4*c*, 4*e*, 4*e*, 4*e*, *A*	6
H 116β	„	„	„	„	„	„	3*c*, 4*c*, *A*	3
H 117a	5*c*	5*c*	*A*	*Wβ*	*A*	*Wβ*	5*c*, 6*e*, *A*	3
H 117β	„	„	„	„	„	„	3*e*, 5*g*, 3*fp*	3
H 118a	3*c*	2*c*	*A*	*Wβ*	*A*	*Wa*	2*bp*, 2*cw*, 3*cw*, 3*c*, 3*e*	5
H 118β	„	„	„	„	„	„	3*bp*, 3*c*, 3*dw*, 3*e*, *A*, *A*, *A*	7

* A ♀ was accidentally fertilised by a ♂ from the same hybrid litter, before the sexes were separated.

† With dark red (not haemoglobin-coloured) eyes, classed as dark-eyed.

TABLE B.—(*continued.*)

Catalogue Number	Parentage						Offspring	Number per Litter
	3	*2*	*7*	*6*	*5*	*4*		
H 119a	5*c*	5*c*	*A*	*Wβ*	*A*	*Wβ*	2*c*, 3*c*, 5*c*, 6*e*, *A*, *Aw*	6
H 119β	"	"	"	"	"	"	*A*, *A*, *A*, *A*, *A*	5
H 120a	5*c*	5*c*	*A*	*Wβ*	*Wa*	*A*	5*bpw*, *Aw*	2
H 120β	"	"	"	"	"	"	4*c*, 4*e*, 5*c*, 5*c*, 5*c*, *A*	6
H 121a	4*c*	4*c*	*Wa*	*A*	*A*	*Wβ*	2*apw*, 2*d*, 3*c*, 4*c*, 4*d*, 4*e*, *A*	7
H 121β	"	"	"	"	"	"	3*bp*, 3*c*, 3*cw*, 4*bp*, 4*bp*, 4*bp*	6
H 122a	3*c*	3*c*	*A*	*W*	*A*	*Wβ*	2*c*, 2*c*, 3*c*, 3*e*, 3*fp*, 4*e*, *A*	7
H 122β	"	"	"	"	"	"	1*c*, 3*ap*, 3*ap*, 4*c*	4
H 123a	5*c*	5*c*	*A*	*Wβ*	*A*	*Wβ*	5*c*, 5*c*, *A*, *A*	4
H 123β	"	"	"	"	"	"	3*c*, 3*e*, 5*c*, 5*bp*, 6*e*, 6*e*	6
H 124a	5*c*	5*c*	*A*	*Wa*	*A*	*Wβ*	3*e*, 4*c*, 5*bp*, 5*cw*, 6*e*	5
H 124β	"	"	"	"	"	"	4*e*, 4*e*, 5*bp*, 5*bp*, *A*	5
H 125	5*c*	5*c*	*A*	*Wβ*	*A* ×	*Wa*	3*c*, 5*bpw*, 5*bc*, 5*c*, 5*c*, 5*c*, 6*c*	7
H 126a	2*d*	2*c*	*A*	*Wa*	*A* ×	*Wβ*	1*cw*, 1*ew*, 2*bp*, 2*bp*, 2*c*, 3*c*, *A*	7
H 126β	"	"	"	"	"	"	1*ap*, 2*ew*, 2*e*, 3*ap*, *A*	5
H 127a	5*c*	5*c*	*A*	*Wβ*	*A* ×	*Wβ*	5*c*, 5*c*, 5*c*, 5*e*, *A*	5
H 127β	"	"	"	"	"	"	5*bpw*, 5*bp*, 5*bp*, 3*c*, 6*g*	5
H 128a	5*c*	5*c*	*A*	*Wβ*	*A*	*W*	4*afp*, 4*e*, 5*afp*, 5*cw*, 6*fpw*, *Aw*, *Aw*	7
H 128β	"	"	"	"	"	"	5*bp*, *A*	2
H 129a	5*c*	5*c*	*A*	*Wβ*	*A*	*W*	5*bp*, 5*c*, 6*e*, 6*fp*	4
H 129β	"	"	"	"	"	"	2*c*, 4*e*, 5*bpw*, 5*cw*, 5*e*, 6*ew*	6
H 130	3*c*	3*c*	*Wa*	*A*	*A*	*W*	1*cw*, 3*bpw*, 4*cw*, *A*, *A*	5
H 133a	5*c*	5*c*	*A* ×	*W*	*A* ×	*W*	2*e*, *A*	2
H 133β	"	"	"	"	"	"	5*c*, 5*c*, 5*c*, *A*, *A*	5
H 134	3*c*	3*c*	*A* ×	*W*	*A* ×	*W*	*A*, *A*	2
H 135a	5*c*	5*c*	*A* ×	*W*	*A* ×	*W*	2*e*, 5*bp*, 6*e*, *Aw*	4
H 135β	"	"	"	"	"	"	5*dw*, *A*	2
H 136a	5*a*	5*c*	*A*	*Wβ*	*A* ×	*W*	2*bp*, 2*c*, 5*a*, 5*a*, 5*cw*	5
H 136β	"	"	"	"	"	"	4*ap*, 5*bp*, 6*ap*	3
H 137a	3*c*	4*c*	*A* ×	*W*	*A* ×	*W*	3*g* (eyes dark red, not blood-colour)	1
H 137β	"	"	"	"	"	"	3*d*, *A*, *A*	3
H 138a	5*c*	5*c*	*A* ×	*W*	*Wa*	*A* ×	3*g*, 5*c*, 5*cw*, *Aw*	4
H 138β	"	"	"	"	"	"	5*bp*, 5*bpw*, 5*c*, 6*gw*, *A*	5
H 139a	5*e*	3*e*	*A*	*Wβ*	*Wa*	*A* ×	1*fp*, 3*fpw*, *A*	3
H 139β	"	"	"	"	"	"	6*c*, *A*	2
H 142a	5*c*	5*c*	*A*	*Wa*	*A*	*W*	6*e*, *A*, *A*	3
H 142β	"	"	"	"	"	"	4*c*, 5*c*, 5*c*, 6*e*, 6*fp*	5
H 145a	2*c*	2*c*	*A* ×	*Wβ*	*A* ×	*W*	2*c*, *A*, *A*, *A*, *Aw*	5
H 145β	"	"	"	"	"	"	1*c*, 1*fpw*, 2*c*, 2*bp*	4
H 148a	5*d*	5*c*	*A*	*Wa*	*A*	*W*	5*c*, 5*c*, 5*c*, 6*e*, *A*, *A*	6
H 148β	"	"	"	"	"	"	4*fp*, 5*c*, *A*, *A*, *Aw*	5
H 150	4*c*	5*c*	*A* ×	*W*	*A*	*Wa*	4*c*, 4*cw*, 5*c*	3
H 201	5*a*	5*a*	*A*	*Wa*	*A*	*Wβ*	3*a*, 5*a*, 5*a*, 5*a*, *A*, *A*, *Aw*	7
H 202	5*a*	5*a*	*A*	*Wa*	*A*	*Wβ*	4*ap*, 5*ap*, *A*, *A*	4
H 210	5*a*	4*c*	*A*	*Wβ*	*A*	*Wβ*	4*bp*, *A*, *A*	3
H 224	3*c*	5*c*	*A* ×	*W*	*A*	*Wa*	1*c*, 2*a*, 2*e*, 3*fp**, 3*fp**, *A*	6
H 225	3*c*	3*c*	*A* ×	*Wa*	*A*	*Wβ*	2*ap*, 3*fp*, 3*c*, 3*c*, 4*e*, *A*, *Aw*	7
H 226	3*c*	4*c*	*A*	*W*	*A* ×	*Wa*	3*bp*, 3*bpw*, 4*ew*, *A*, *A*, *A*	6
H 227	4*c*	2*c*	*A*	*W*	*A*	*Wβ*	2*cw*, 2*e*, 3*ew*, 4*bpw*, 4*bpw*, 4*fp*	6
H 228	3*c*	4*c*	*A* ×	*W*	*A* ×	*W*	2*c*, 3*c*, 3*d*, *A*, *A*, *Aw*	6
H 229	2*c*	3*c*	*A* ×	*Wβ*	*Wβ*	*A*	2*fp**, 2*c*, 2*fpw*, 3*fpw*, 3*c*, 4*c*	6
H 230	3*c*	3*c*	*Wβ*	*A*	*Wa*	*A*	4*bp*	1
H 231	3*c*	3*c*	*A*	*W*	*A* ×	*Wa*	3*bc*, 3*c*, 3*c*, 3*d*, 3*e*, 4*e*, *A*, *A*	8
H 232	3*d*	4*c*	*Wa*	*A*	*A*	*Wβ*	1*bpw*, 2*bp*, 3*bp*, 3*c*, 4*e*, *A*	6
H 233	3*c*	1*c*	*A*	*Wβ*	*A*	*W*	2*c*, 2*c*, 2*c*, 2*c*, 2*c*, 2*c*, 2*ew*, *A*, *A*	9
H 234	2*e*	5*c*	*Wa*	*A*	*A*	*W*	2*fpw*, 3*c*, 5*cw*, 6*fp*, 6*fp*	5
H 235	2*c*	5*c*	*A*	*Wβ*	*A*	*Wβ*	2*c*, 3*e*, 4*dw*, 5*c*, *A*, *A*	6
H 262	5*d*	2*c*	*A*	*Wa*	*A*	*W*	2*bpw*, 3*bp*, 3*e*, 5*bp*, 5*c*, *A*	6
							Total	555

* The record of eye-colour in these mice is uncertain.

TABLE C.—*Young Produced by pairing Hybrids and Albinos.*

Catalogue Number	Parentage						Coloured Young	Albinos	Number in Litter
	3	*2*	*7*	*6*	*5*	*4*			
H 1	*A*	*3c*	*A*	*A*	*A* ×	*Wa*	*5c*, *5c*, *5c*, *5c*	—	4
H 2	*A*	*3c*	*A*	*A*	*A* ×	*Wa*	*4d*, *4d*, *4e*, *5d*	5	9
H 3	*A*	*2d*	*A*	*A*	*A* ×	*Wa*	*5c*, *5c*, *5c*, *5c*, *6c*, *6d*	4	10
H 4	*A*	*5c*	*A*	*A*	*A*	*W*	*5c*, *5d*, *6e*	2	5
H 5	*A*	*5c*	*A*	*A*	*A*	*W*	*6c*, *6d*, *6d*, *6d*	1	5
H 6	*A*	*5c*	*A*	*A*	*A*	*Wa*	*6d*, *6d*	2	4
H 11	*A*	*3c*	*A*	*A*	*A*	*Wa*	*6e*	5	6
H 12	*3c*	*A*	*A*	*W*	*A*	*A*	*3c*, *6e*, *6e*	2	5
H 13	*5c*	*A*	*A*	*W*	*A*	*A*	*5d*, *6e*	1	3
H 14	*5c*	*A*	*A*	*W*	*A*	*A*	*2e*, *3e*	4	6
H 15	*A*	*5c*	*A*	*A*	*A*	*W*	*3a*, *3a*, *3e*, *6a*	4	8
H 16	*A*	*5c*	*A*	*A*	*A*	*Wa*	*3e*, *4g*	5	7
H 17	*A*	*5c*	*A*	*A*	*A*	*W*	*3d*, *3e*, *6d*	2	5
H 24	*A*	*5d*	*A*	*A*	*A*	*Wa*	*4d*, *4d*, *4d*	3	6
H 26	*A* ×	*3c*	*A* (?)*	*A* (?)*	*A*	*Wa*	*5d*, *5d*, *6e*	1	4
H 27	*3c*	*A*	*A*	*W*	*A*	*A*	*6e*	5	6
H 28a	*5c*	*A*	*A*	*Wa*	*A*	*A*	*5c*, *5d*	—	2
H 28β	,,	,,	,,	,,	,,	,,	*5d*, *6d*	3	5
H 31	*A*	*5c*	*A*	*A*	*A*	*Wa*	— — —	1	1
H 32	*2c*	*A*	*A*	*Wa*	*A*	*A*	*4c*, *6c*, *6c*, *6e*	3	7
H 33	*A*	*5c*	*A*	*A*	*A*	*W*	*6e*, *6e*, *6e*, *6e*	4	8
H 34	*3c*	*A*	*A* ×	*Wβ*	*A*	*A*	*5c*, *5c*, *6d*	1	4
H 35	*3c*	*A*	*A* ×	*Wβ*	*A*	*A*	*4d*, *5d*, *5d*, *5d*, *5d*	1	6
H 38	*2d*	*A*	*A*	*Wa*	*A*	*A*	*5d*, *5d*	2	4
H 39	*2d*	*A*	*A*	*Wa*	*A*	*A*	— — —	4	4
H 40	*5a*	*A*	*A*	*Wa*	*A*	*A*	*6a*, *6a*	5	7
H 41	*5c*	*A*	*A*	*Wa*	*A*	*A*	*5c*, *6e*	3	5
H 43	*A*	*2c*	*A* ×	*Wa*	*A*	*A*	*5c*, *6e*, *6e*	2	5
H 45	*A* ×	*5c*	*A* (?)†	*A* (?)†	*A*	*W*	*6e*	3	4
H 46	*A*	*2c*	*A*	*A*	*A*	*Wa*	*3c*, *3c*, *3c*	5	8
H 47	*A* ×	*3c*	*A* (?)†	*A* (?)†	*A*	*Wa*	*6e*, *6e*	4	6
H 48	*A*	*5c*	*A*	*A*	*A*	*Wa*	*6e*	3	4
H 49a	*A*	*5d*	*A*	*A*	*A*	*Wa*	*6e*, *6g*	3	5
H 49β	,,	,,	,,	,,	,,	,,	*3c*, *5d*, *6e*	2	5
H 51	*A* ×	*5c*	*A* (?)†	*A* (?)†	*A*	*Wa*	*4c*, *4c*, *4e*	1	4
H 52	*A*	*3d*	*A*	*A*	*A*	*Wa*	*6e*, *6e*	3	5
H 55	*2c*	*A*	*A* ×	*Wa*	*A*	*A*	*3a*, *6g*	4	6
H 56	*2c*	*A*	*A* ×	*Wa*	*A*	*A*	*4c*, *4e*, *6d*	1	4
H 57	*A* ×	*5c*	*A* (?)†	*A* (?)†	*A*	*W*	*4c*, *4c*, *4c*, *5d*	3	7
H 58	*2c*	*A* ×	*A* ×	*Wβ*	*A* ×	*A* ×	*2a*	3	4
H 59	*2c*	*A*	*A*	*Wa*	*A*	*A*	*5c*, *6e*	5	7
H 60	*A*	*3c*	*A*	*A*	*A*	*Wa*	*4c*, *4c*, *6e*, *6e*	2	6
H 63	*3c*	*A*	*A* ×	*Wa*	*A*	*A*	*2g*, *5d*, *6e*	2	5
H 64	*2c*	*A* ×	*A* ×	*Wa*	*A* ×	*A*	*2d*, *2e*	2	4
H 65	*2e*	*A*	*Wa*	*A*	*A*	*A*	*6e*	3	4
H 80	*A*	*3c*	*A*	*A*	*A* ×	*Wa*	*5g*, *6e*	5	7
H 81	*5c*	*A* ×	*A*	*Wβ*	*A*	*6e*	*3e*, *4e*, *6e*, *6e*, *6e*	4	9
H 83	*5e*	*A* ×	*A*	*Wβ*	*A*	*6e*	*4e*, *5e*, *5e*, *6e*	1	5
H 85	*5c*	*A*	*A*	*Wβ*	*A*	*A*	— — —	4	4
H 86	*5c*	*A* ×	*A*	*Wβ*	*A*	*6e*	*5c*, *5c*	2	4
H 87	*5c*	*A* ×	*A*	*Wβ*	*A*	*6e*	*6c*, *6e*	3	5
H 88a	*A*	*3c*	*A* ×	*A* ×	*A*	*Wa*	*5c*, *5c*, *6e*	3	6
H 88β	,,	,,	,,	,,	,,	,,	*6e*	5	6
H 92	*A*	*5c*	*A*	*A*	*A*	*W*	*3c*, *4e*, *6e*	3	6
H 131	*A*	*2c*	*A*	*A*	*A*	*W*	*2e*, *3c*, *3c*	1	4
H 140	*5c*	*A*	*A* ×	*W*	*A*	*A*	*3c*, *6c*	4	6
H 143a	*A*	*5c*	*A*	*A*	*A*	*W*	*5c*, *6c*, *6c*	3	6
H 143β	,,	,,	,,	,,	,,	,,	*4c*, *5c*	2	4
H 144a	*A*	*4c*	*A*	*A*	*A* ×	*W*	*2e*	1	2
H 144β	,,	,,	,,	,,	,,	,,	*5c*	3	4
H 146a	*2c*	*A*	*A* ×	*Wβ*	*A*	*A*	*5c*, *6c*, *6e*	3	6
H 146β	,,	,,	,,	,,	,,	,,	*5c*, *5c*, *5c*	2	5
H 147	*3c*	*A*	*A* ×	*Wβ*	*A*	*A*	*3c*	4	5
H 149a	*5d*	*A*	*A*	*Wa*	*A*	*A*	*5c*, *5c*	4	6
H 149β	,,	,,	,,	,,	,,	,,	*5d*, *5d*, *6d*	3	6
H 151a	*3c*	*A*	*A* ×	*W*	*A*	*A*	*5c*, *5c*, *6c*, *6c*	1	5
H 151β	,,	,,	,,	,,	,,	,,	*5c*, *6c*	3	5

* Pedigree of these albinos uncertain.

† Pedigree unknown.

TABLE C.—(*continued.*)

LIBRARY OF THE UNIVERSITY OF WYOMING LARAMIE.

Catalogue Number	Parentage						Coloured Young	Albinos	Number in Litter
	3	*2*	*7*	*6*	*5*	*4*			
H 152	3*c*	*A*	*A*	*W*	*A*	*A*	6*c*	5	6
H 153	3*c*	*A*	*A*	*W*	*A*	*A*	3*e*, 4*bc*	3	5
H 154	3*c*	*A*	*A*	*W*	*A*	*A*	5*bc*, 6*e*, 6*e*	2	5
H 155	2*c*	*A*	*A*	*Wβ*	*A*	*A*	4*c*, 4*c*, 5*c*, 6*e*	2	6
H 156	3*c*	*A*	*A*	*Wβ*	*A*	*A*	6*e*	3	4
H 157	5*a*	*A*	*A*	*Wβ*	*A*	*A*	5*a*, 5*a*, 5*d*	3	6
H 158α	5*a*	*A*	*A*	*Wβ*	*A*	*A*	5*c*, 5*c*, 5*c*	3	6
H 158β	,,	,,	,,	,,	,,	,,	5*a*, 5*a*, 5*c*, 5*c*, 5*c*	1	6
H 159	5*a*	*A*	*A*	*Wβ*	*A*	*A*	5*c*	2	3
H 160	5*a*	*A*	*A*	*Wa*	*A* ×	*A*	5*c*, 5*c*, 5*c*, 5*c*	2	6
H 161	5*a*	*A*	*A*	*Wa*	*A*	*A*	5*a*, 5*c*, 5*c*	2	5
H 162	5*a*	*A*	*A*	*Wa*	*A* ×	*A*	5*a*, 5*c*	1	3
H 163	5*a*	*A*	*A*	*Wa*	*A*	*A*	5*a*, 5*b*, 4*e*, 4*c*	—	4
H 164α	*A*	4*c*	*A* ×	6*e*	*A* ×	*Wa*	5*c* (pale), 6*e*	1	3
H 164β	,,	,,	,,	,,	,,	,,	6*e*, 6*e*	5	7
H 165	*A* ×	3*c*	*A* ×	6*e*	*A*	*Wβ*	5*c*	1	2
H 166	*A* ×	2*c*	*A* ×	6*e*	*A* ×	*Wa*	5*c*, 5*c*, 5*d*	5	8
H 167	*A* ×	5*c*	*A* ×	6*e*	*A*	*Wa*	5*d*, 6*c*, 6*c*, 6*d*, 6*e*	3	8
H 168	5*a*	*A* ×	*A*	*Wβ*	*A* ×	6*e*	4*a*, 5*a*, 5*a*, 6*c*	3	7
H 169	3*c*	*A* ×	*A* ×	*Wβ*	*A* ×	6*e*	4*c*, 5*c*, 5*c*, 6*c*, 6*e*	3	8
H 170	*A* ×	5*c*	6*g*	6*e*	*A*	*Wβ*	5*c* (pale), 5*c*, 6*g*	2	5
H 171	5*c*	*A* ×	*A* ×	*Wa*	6*g*	6*e*	6*c*, 6*c*, 6*c*	2	5
H 172	3*c*	*A* ×	*A* ×	*Wa*	*A* ×	6*e*	4*c*, 4*c*, 6*e*	2	5
H 173	5*c*	*A*	*A*	*Wβ*	*A*	*A*	5*c*, 6*e*, 6*e*	3	6
H 175	5*c*	*A*	*A*	*Wβ*	*A*	*A*	5*b*, 5*bc*, 5*c*, 5*d*, 4*e*	2	7
H 178	5*c*	*A*	*A* ×	*W*	*A*	*A*	3*c*, 5*c*, 5*c*	5	8
H 179	5*c*	*A*	*A*	*Wa*	*A*	*A*	5*c*, 5*c*, 5*c*, 5*d*, 6*d*	1	6
H 180	5*c*	*A*	*A*	*Wa*	*A*	*A*	5*c*, 5*c*, 5*c*, 5*c*, 6*c*	2	7
H 181	5*c*	*A*	*A* ×	*W*	*A*	*A*	5*c*, 5*e*, 6*e*	3	6
H 182	5*c*	*A*	*A*	*Wa*	*A*	*A*	5*c*, 5*c*, 6*e*	3	6
H 183	5*c*	*A*	*A*	*Wβ*	*A*	*A*	5*c*, 5*c*	4	6
H 184	5*c*	*A*	*A*	*Wβ*	*A*	*A*	6*e*	8	9
H 185	5*c*	*A*	*A*	*W*	*A*	*A*	4*e*, 5*c*, 6*e*, 6*e*, 6*e*, 6*e*, 6*e*	2	9
H 186	*A*	5*c*	*A* ×	*A*	*A*	*Wβ*	5*c*, 5*d*	6	8
H 188	*A*	5*c*	*A* ×	*A*	*Wa*	*A* ×	5*e*, 5*g*	—	2
H 189	*A*	5*c*	*A* ×	*A*	*A*	*Wβ*	6*c*, 3*e*, 2*d*, 6*e*	3	7
H 191	*A*	5*c*	*A*	*A*	*A*	*Wβ*	3*c*, 6*e*, 6*e*	2	5
H 192	*A*	5*c*	*A*	*A*	*A*	*Wβ*	5*c*, 5*c*, 5*d*, 4*e*, 6*e*	2	7
H 193	*A*	5*c*	*A*	*A*	*A* ×	*Wβ*	6*e*	1	2
H 194	*A*	5*d*	*A*	*A*	*A*	*Wa*	3*e*, 5*c*, 5*d*, 6*e*	5	9
H 195	*A*	5*c*	*A*	*A*	*A*	*Wβ*	5*c*, 5*c*, 5*c*, 5*d*, 6*e*	3	8
H 196	*A*	5*c*	*A*	*A*	*A*	*Wa*	6*d*, 6*d*, 6*d*, 6*d*	5	9
H 197	*A*	5*c*	*A*	*A*	*A* ×	*Wa*	5*c*, 5*c*	4	6
H 198	*A*	5*c*	*A*	*A*	*A*	*Wβ*	4*c*, 4*e*, 5*c*, 5*c*, 6*c*, 6*e*, 6*e*	2	9
H 199	*A*	5*c*	*A*	*A*	*A*	*Wβ*	4*e*, 5*c*	5	7
H 200	*A*	5*c*	*A*	*A* ×	*A* ×	*W*	3*bc*	4	5
H 203	5*a*	*A*	*A*	*Wa*	*A*	*A*	5*a*, 5*a*, 5*c*, 5*c*	2	6
H 204	5*a*	*A*	*A*	*Wa*	*A*	*A*	5*a*, 5*a*, 5*c*, 5*c*	3	7
H 205	5*a*	*A*	*A*	*Wa*	*A*	*A*	5*a*, 5*a*, 5*c*, 5*c*, 5*c*, 5*c*	—	6
H 206	*A*	5*c*	*A*	*A*	*A* ×	*W*	4*c*, 4*c*, 4*c*, 5*c*, 5*c*, 6*e*	3	9
H 207	*A*	5*c*	*A*	*A*	*A* ×	*W*	4*c*, 4*d*, 5*c*, 6*e*, 6*e*, 6*e*	3	9
H 208	5*c*	*A*	*A*	*Wβ*	*A*	*A*	4*c*, 6*g*	5	7
H 209	5*a*	*A*	*A*	*Wβ*	*A*	*A*	5*bc*	1	2
H 212	*A*	2*c*	*A* ×	*A*	*A* ×	*Wβ*	3*e*, 6*e*, 6*e*	4	7
H 213	*A*	3*c*	*A* ×	*A*	*A*	*Wβ*	3*bc*, 3*c*, 4*c*, 4*c*, 4*c*	2	7
H 214	*A*	3*c*	*A* ×	*Ap*	*Ap*	*Wa*	3*e*, 4*e*, 6*e*, 6*d*	4	8
H 216	*A*	3*c*	*A*	*A*	*A*	*Wβ*	3*g*, 4*e*	—	2
H 217	4*d*	*A*	*A* ×	*Wa*	*A* ×	*A*	3*bc*, 3*c*, 4*g*	3	6
H 218	2*c*	*A*	*A* ×	*Wβ*	*A* ×	*A*	3*bc*, 3*c*, 4*e*, 4*e*	3	7
H 219	5*d*	*A*	*A*	*Wa*	*A* ×	*A*	5*c*, 6*e*, 6*e*	3	6
H 220	2*c*	*A*	*A* ×	*Wa*	*A* ×	*A*	5*c*, 5*c*, 5*c*	2	5
H 222	4*c*	*A*	*A* ×	*W*	*A*	*A*	5*bc*, 5*bc*, 3*c*, 3*c*, 5*c*	4	9
H 223	2*c*	*A*	*A* ×	*Wβ*	*A*	*A*	4*g*, 5*c*	1	3
H 260	4*c*	*A* ×	*A* ×	*W*	6*e*	*A* ×	— — —	1	1
H 261	4*c*	*A*	*A* ×	*W*	6*e*	*A* ×	4*c*, 5*c*	5	7
								Total	746

TABLE D.

Results of mating Hybrids with "Extracted Albinos."

(*H = Hybrid, E = Extracted Albino; see text, p.* 25.)

Catalogue Number	Parents of Hybrid	Parents of Albino	Offspring	Number in Litter
G 1	*A* × *W*	*H* × *H*	3*ap*, 3*bp*, 5*aw*, *A*, *A*, *Aw*	6
G 2	*A* × *W*	*H* × *H*	1*bc*, 1*bc*, 3*c*, *A*	4
G 5	*A* × *W*	*H* × *H*	5*c*, 5*c*, 5*c*, 5*c*, *A*, *A*, *Aw*	7
G 6	*A* × *W*	*H* × *H*	2*c*, 3*c*, 3*c*, *A*, *A*, *A*	6
G 7	*A* × *W*	*H* × *H*	5*bp*, *A*, *A*, *A*, *Aw*	5
G 8	*A* × *W*	*H* × *H*	5*c*, 5*cw*, 6*e*, *A*, *A*	5
G 9	*A* × *W*	*H* × *H*	5*bp*, 5*c*, 5*c*, *A*, *A*	5
G 10	*A* × *W*	*H* × *H*	3*c*, 3*e*, 4*e*, *A*, *Aw*	5
G 11	*A* × *W*	*H* × *H*	5*cw*, *A*, *A*, *A*, *A*	5
G 3	*A* × *W*	*H* × *A*	*A*, *A*, *A*	3
G 4	*A* × *W*	*H* × *A*	5*a*, 5*a*, 5*aw*, 4*c*, *A*, *A*, *Aw*	7
G 22	*A* × *W*	*H* × *A*	5*c*, 5*c*, 6*e*, *A*	4
G 23	*A* × *W*	*H* × *A*	4*c*, 5*c*, *A*, *Aw*	4
G 24	*A* × *W*	*H* × *A*	5*c*, 6*e*, *A*, *A*	4
G 25	*A* × *W*	*H* × *A*	6*e*, 6*g*, *A*, *A*, *A*, *Aw*	6
G 26	*A* × *W*	*H* × *A*	2*c*, 3*c*, 5*c*, 6*e*, *A*, *A*, *A*	7
G 27	*A* × *W*	*H* × *A*	3*c*, 4*c*, 5*cw*	3
G 12	*E* × *W*	*H* × *A*	3*bp*, 3*d*, 3*gw*, *A*, *A*	5
G 14	*E* × *W*	*H* × *A*	4*c*, 5*c*, 5*c*, *A*, *A*	5
G 15	*E* × *W*	*H* × *A*	5*a*, *A*, *A*, *A*, *A*	5
G 16	*E* × *W*	*H* × *A*	5*a*, 5*a*, 5*a*, 5*a*, 3*e*, *A*, *Aw*	7
G 17	*E* × *W*	*H* × *A*	3*c*	1
G 19	*E* × *W*	*H* × *A*	3*e*, *A*, *A*, *A*, *A*	5
G 20	*E* × *W*	*H* × *A*	3*bp*, 3*bpw*, 5*c*, *A*, *A*, *A*	6
			Total	120

TABLE E.—*Result of pairing "Extracted Hybrids."*

	Catalogue Number	Parentage 3	2	7	6	5	4	Offspring	Number in Litter
Unions of Type (H × H) × (H × H)	2H 36	5b	3e	5a	5a	6d	5c	3a, 5a, 5c	3
	2H 39a	5c	1e	5c	5c	2c	2c	1c, 2c, 2c, 2c, 5c, 5bp, 5bp	7
	2H 39β	„	„	„	„	„	„	2c, 3c, 4c, 5bp, 5c	5
	2H 46	4d	5c	3c	4c	5c	5c	6bp	1
	2H 48	5a	1c	5a	5c	4c	3c	4ap, 5a, 5a, A, A	5
	2H 49	6e	3c	5c	5c	5c	5c	5d, 5d, 6e, 6e, 6e, 6e	6
	2H 50a	4e	3c	4c	3c	3c	4c	2c, 3c, 3c, 4e, A	5
	2H 50β	„	„	„	„	„	„	2c, 3e, 4e, 4e, A	5
	2H 51	4c	2c	4c	4c	3c	4c	1c, 3c, 3c, 3e, 3g	5
	2H 52	3c	5d	3c	5c	5c	5c	3c, 5c, 5c, 5bp, A	5
	2H 53a	5d	3c	6d	5c	3c	2c	3e, 3e, 4e, 4e, 5d, 6e, 6e, Aw	8
	2H 53β	„	„	„	„	„	„	3bp, 3c, 3c, 5cw, 6e	5
	2H 92	3e	4c	4c	3c	4c	5c	3c, 3cw, 4c	3
	2H 93	4e	4c	4c	3c	3c	5c	2fp, 3c, 3e, 4fp, A	5
	2H 96	6e	4c	5c	5c	4c	3c	3e, 3g, 5c, 5d, 5d, 6e, A	7
	2H 99	4bc	5c	4c	4c	5c	5c	4bc, 4bc, 5c, A, A	5
	2H 139	5a	6c	5a	5c	3c	5c	5a, 5b, 5cw, 6e, 6fpw	5
	2H 142	5c	5c	3c	5c	5d	5c	5bpw, 5bcp, 5bc, 5bc, 5c, 5c, 5c, 5cw	8
Unions of Type (H × A) × (H × H)	2H 23	5c	5c	5a	A	5d	5c	5bp, 5c, 5c, A, A	5
	2H 28	5d	4e	5a	A	3c	5c	5bp, 5c, 5c, 6e, 6e	5
	2H 29	6e	6c	3c	A	3c	5c	5c, 5c, 6e, A	4
	2H 32	6d	5c	A	2c	5a	5a	4c, 5c, 5c, 5c, 5c	5
	2H 33	5c	6b	A	2c	5a	5a	4bpw, 5b, 5b, 5c, 5c, 5cw	6
	2H 34	6d	3e	A	2c	5c	5c	3bp, 3c, 3c, 4c, 6e, 6e, A	7
	2H 38	6e	2c	2e	A	2c	2c	3bc, 3bpw, 5bp, 5bp, 5d, A	6
	2H 40	5c	3c	5c	5c	A	3c	5bc, 5bc, 5c	3
	2H 41	6e	6c	2e	A	5c	5c	3c, 6e, 6e, 6e, A	5
	2H 43	6e	3e	5c	A ×	4c	4c	3e, 3e, 3fp, 3fp, 4fp, 4e, 6e, 6e	8
	2H 44a	5c	2c	3c	A	3c	5c	3ap, 3c, 3c, 4c, 5bc, 5c, A	7
	2H 44β	„	„	„	„	„	„	2c, 5bc, 5c, A, A	5
	2H 100	5a	5d	5a	A ×	5c	5c	A, A, A	3
	2H 101	5a	6e	5a	A ×	5c	5c	5a, 6c, 6c, 6c, 6cw, A, A	7
	2H 103	5c	3e	5a	A	5c	5c	3c, 4e, 4e, 5c, 5c	5
	2H 106	5d	4e	A ×	3c	5d	5c	2e, 4d, A, A	4
	2H 107	6e	2c	A ×	3c	3c	2c	3c, 3c, 3fp, 3fp, 5bp, 5d	6
	2H 118	1c	6e	3c	4c	4c	A ×	3b, 5b, 5c, 5c, 5c	5
	2H 119	5c	6e	5c	5c	A ×	5c	4e, 5d, 6e, 6e, A, A	6
	2H 145	6e	5c	3e	A	5c	5c	3c, 4c, 5c, A, A	5
Unions of Type (H × A) × (H × A)	2H 18	6c	6e	3c	A	3c	A	6bcp, 6d, 6d, 6fp, A, A	6
	2H 19	6e	5c	3c	A	5a	A	3c, 3d, 5c, 5c, A	5
	2H 20	5c	5a	3c	A	5a	A	4a, 5a, 5a, 5d, 6d, A, A	7
	2H 22	5c	3e	3c	A	5c	A	4c, 4c, 5c, 5c, A	5
	2H 25	5c	6e	5a	A	A	3c	6e, A, A, A	4
	2H 26	5a	4c	5a	A	A	3c	3c, 4a, 4c, 5a, A, A	6
	2H 27	5a	6e	5a	A	A	3c	5bc, 5apw, 5a, 6ew, 3e	5
	2H 37	6e	3e	A ×	3c	A	3c	4e, 4e, 6e, A, A, A	6
	2H 45	5g	5c	A	3c	A	5c	4c, 5c, 5c, A, A, A	6
	2H 110	6e	5d	3c	A ×	5d	A	6d, 6d, 6dw	3
	2H 111	6c	5a	3c	A ×	5a	A	4a, 4a, 4e, 5bp	4
	2H 112	4c	5a	A	5c	5a	A	4a, 4e, 4e, 4e, 4d, 5a, 5a, 6e	8
	2H 113	6c	5c	3c	A	5a	A	3c, 5c, A, A, A	5
	2H 114	5d	6c	3c	A	3c	A ×	4d, 4d, 6d, 6e	4
	2H 115	5c	6d	3c	A	3c	A ×	4e, 4e, 5d, A	4
	2H 116	5d	5c	5d	A	3c	A ×	5bp, 5bp, 5bp, 5bp, A, A	6
	2H 143	6e	6e	2e	A	A	3c	2fpw, 3e, 6e, 6e, 6e, 6ew, 6fpw	7
	2H 144	5a	4g	5a	A	5c	A	3e, 4e, 6g, 6g, A, A	6
	2H 146	5c	5c	5a	A	A	5c	4bp, 5bp, 3c, 5c, 6e, 6fp, A	7
	2H 147	5a	6e	5a	A	5c	A	3a, 3ap, 5a, 6g, A, A, A, A	8
	2H 148	5c	5c	3c	A	5a	A	3c, 3c, 5c, 5c, 4bcp, 4bp, 4ap, 5ap, 5ap	9
								Total	321

TABLE F.

Result of mating "Extracted Hybrids" and "Extracted Albinos."

Catalogue Number	Parents of Albino	Parents of Hybrid	Offspring	Number in Litter
K 1	*H* × *H*	*H* × *A*	6*e*, 5*d*, 5*d*	3
K 2	*H* × *A*	*H* × *H*	4*c*, 5*c*, 5*c*, 5*d*, *A*	5
K 3	*H* × *H*	*H* × *H*	5*c*, 5*c*, 5*e*	3
K 4	*H* × *H*	*H* × *H*	5*c*, *A*, *A*, *A*	4
K 5	*H* × *H*	*H* × *H*	5*c*, 5*c*, 5*c*, *A*, *A*, *Aw*	6
K 6	*H* × *H*	*H* × *H*	5*c*, 5*c*, *A*, *A*, *A*, *A*, *Aw*	7
K 10	*H* × *H*	*H* × *H**	3*bpw*, 4*b*, 4*bpw*	3
K 11	*H* × *A*	*H* × *H*	6*bp*, 6*e*, 6*fp*, *A*	4
K 12	*H* × *H*	*H* × *A*	3*bc*, 3*c*, 3*c*, *A*, *A*	5
K 15	*H* × *H*	*H* × *H*	3*bp*, 6*bp*, *A*, *A*, *A*	5
K 16	*H* × *H*	*H* × *H*	2*e*, 3*e*, 3*e*, 3*c*, 4*c*	5
K 19	*H* × *A*	*H* × *H*	3*c*, 4*c*, *A*	3
K 21	*H* × *A*	*H* × *H*	4*e*, 4*e*, 4*e*, 6*d*, 6*e*	5
K 23	*H* × *H*	*H* × *A*	4*c*, 5*c*, 5*c*, 5*c*, *A*, *A*, *A*, *A*	8
K 24	*H* × *H*	*H* × *A*	1*e*, 2*c*, 2*c*, *A*, *A*, *A*	6
K 27	*H* × *A*	*H* × *H*	6*e*, 6*e*, *A*, *A*, *A*	5
K 28	*H* × *H*	*H* × *A*	4*c*, 4*c*, 5*c*, 6*g*, *A*, *A*, *A*	7
K 29	*H* × *A*	*H* × *H*	2*d*, 3*c*, 3*e*, *A*, *A*, *A*	6
K 30	*H* × *A*	*H* × *H*	*A*	1
			Total	91

* The eyes of the hybrid used (the female) were possibly dark red, not black.

TABLE G.

Result of mating "Extracted Dominants" and Albinos. (E = "Extracted Albino.")

Catalogue Number	Parentage						Offspring	Number in Litter
	3	*2*	*7*	*6*	*5*	*4*		
2H 79	*A*	3*bp*	*A*	*A*	3*c*	2*c*	3*c*, 3*c*, 4*c*, 5*c*, 3*e*, 3*e*, 6*e*, 6*e*	8
2H 80	*A*	3*fp*	*A*	*A*	3*c*	4*c*	3*e*, 6*e*	2
2H 88	4*bp*	*A*	4*c*	3*c*	*A*	*A*	4*bc*, 4*bc*, 4*bc*, *A*	4
2H 90	5*bp*	*E*	5*a*	5*a*	5*a*	*A*	5*b*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6
2H 102	*E*	6*bp*	5*a*	*A* ×	5*c*	5*c*	5*a*, 5*a*, 5*a*, 5*c*, 5*c*, 6*e*, 6*e*	7
2H 104	*E*	2*bp*	*A* ×	3*c*	3*c*	3*c*	5*c*, 5*cw*	2
2H 105	*E*	2*bp*	*A* ×	3*c*	3*c*	3*c*	3*c*, 3*c*, 3*c*, 4*c*	4
2H 108	*E*	2*fp*	5*c*	*A* ×	6*e*	3*e*	3*cw*, 4*e*, 4*e*, 5*c*, 5*c*, 6*e*, 6*e*, 6*e*	8
2H 109	*E*	2*bp*	5*c*	*A* ×	4*c*	3*c*	4*c*, 4*c*, 4*c*, 4*c*, 4*cw*, 4*e*	6
2H 120	*E*	6*bp*	3*c*	*A*	5*c*	5*c*	5*c*, 5*c*, 5*bp*	3
2H 123	*E*	1*bp*	3*c*	*A* ×	5*c*	5*d*	2*e*, 4*c*, 4*e*, 5*c*, 5*c*, 6*e*	6
2H 124	4*ap*	*E*	3*c*	4*c*	3*c*	*A*	4*cw*, 6*e*, 6*e*, *A*, *A*	5
2H 125	*E*	3*bp*	5*d*	*A*	5*c*	5*c*	2*c*, 3*bc*, 4*c*, 5*c*, 5*c*, 5*c*	6
2H 126	4*ap*	*E*	5*a*	5*c*	4*c*	*A* ×	3*c*, 3*c*, 3*c*, 4*a*, 5*d*	5
2H 127	5*bp*	*E*	5*a*	5*c*	3*c*	*A* ×	5*c*, 5*c*, 4*e*, 6*d*, 6*e*	5
cr. 145	*A*	3*bp*	*A*	*A*	5*a*	3*c*	5*b*, 5*b*, 5*b*, 6*b*, 6*b*, 5*c*	6
cr. 146	—	—	—	—	—	—	4*c*, 5*a*, 5*c*, 5*c*, *A*, *A*, *A*, *A*	8
cr. 155	—	—	—	—	—	—	5*a*, *A*, *A*	3
cr. 156	—	—	—	—	—	—	5*bc*, *A*, *A*, *A*	4
							Total	98

TABLE H.

Result of pairing "Extracted Dominants."

Catalogue Number	Parentage						Offspring	Number in Litter
	3	*2*	*7*	*6*	*5*	*4*		
2H 11	2*bp*	3*ap*	3*c*	3*c*	3*d*	2*c*	2*bp*, 2*bp*	2
2H 12	5*bp*	4*ap*	5*c*	5*c*	5*a*	3*c*	2*bp*, 4*bp*, 5*bp*, 5*bp*, 5*bcp*	5
2H 13	6*fp*	3*fp*	5*c*	5*c*	3*c*	4*c*	6*fp*, 6*fp*, 6*fp*, 6*fp*	4
2H 15	2*bp*	4*bpw*	5*a*	5*c*	5*c*	5*c*	3*bp*, 3*bp*, 3*bp*, 4*bp*, *A*	5
2H 55	2*bp*	6*bp*	3*c*	2*c*	5*c*	5*c*	1*ap*, 2*ap*, 2*ap*, 5*ap*	4
2H 85	3*fp*	3*ap*	5*c*	5*c*	5*c*	3*c*	3*bp*, 3*bp*, 3*bp*, 3*bp*, 3*bp*, 3*bp*	6
2H 135	6*fp*	6*bp*	5*c*	5*c*	5*c*	5*c*	5*bp*, 5*bp*, 5*bp*, 5*bp*, 5*bp*, 5*bp*	6
							Total	32

APPENDIX II.

TABLES OF ORGANIC, FRATERNAL, AND ANCESTRAL CORRELATIONS.

TABLE I.

Organic Correlation: Whiteness and Coat-Colour in Hybrids.

Whiteness.

Coat-Colour.	*1*	*2*	*3*	*4*	*5*	*6*	Totals
a	—	—	—	—	12	—	12
c	4	28	57	37	164	2	292
d	—	9	3	1	11	2	26
e	—	2	4	1	3	—	10
Totals	4	39	64	39	190	4	340

TABLE II.

Fraternal Correlation in Whiteness among Hybrids.

First Brother.

Second Brother.	*1*	*2*	*3*	*4*	*5*	*6*	Totals
1	2	8	7	2	1	—	20
2	8	74	45	10	49	—	186
3	7	45	82	47	136	2	319
4	2	10	47	62	99	1	221
5	1	49	136	99	816	3	1104
6	—	—	2	1	3	—	6
Totals	20	186	319	221	1104	6	1856

TABLE III.

Fraternal Correlation in Colour among Hybrids.

First Brother.

Second Brother.	*a*	*c*	*d*	*e*	Totals
a	42	26	25	—	93
c	26	1522	28	12	1588
d	25	28	66	15	134
e	—	12	15	14	41
Totals	93	1588	134	41	1856

TABLE IV.

Whiteness in Waltzing Parents and in Hybrids.

Hybrids.

Waltzing Parents.	*1*	*2*	*3*	*4*	*5*	*6*	Totals
α	2	24	29	9	46	3	113
β	—	8	14	12	62	1	97
Totals	2	32	43	21	108	4	210

TABLE V.

Whiteness in Waltzing Parents and Colour in Hybrids.

Hybrids.

Waltzing Parents.	*a*	*c*	*d*	*e*	Totals
α	7	76	25	5	113
β	5	86	1	5	97
Totals	12	162	26	10	210

TABLE VI.

Purity of Albino Parent and Whiteness in Hybrids.

Hybrids.

Albino Parent.	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Pure bred ...	2	21	34	23	129	3	212
Cross bred ...	2	18	30	16	61	1	128
Totals	4	39	64	39	190	4	340

TABLE VII.

Purity of Albino Parent and Colour of Hybrids.

Albino Parent.	Hybrids.				
	a	*c*	*d*	*e*	Totals
Pure bred ...	12	174	20	6	212
Cross bred ...	—	118	6	4	128
Totals	12	292	26	10	340

TABLE VIII.

Correlation in Coat-Colour between ♂ and ♀ Hybrids mated.

Female.	Male.				
	a	*c*	*d*	*e*	Totals
a	24	20	—	—	44
c	—	416	15	—	431
d	—	56	14	—	70
e	—	5	—	5	10
Totals	24	497	29	5	555

TABLE IX.

Correlation in Amount of Whiteness between ♂ and ♀ Hybrids mated.

Female.	Male.						
	1	*2*	*3*	*4*	*5*	*6*	Totals
1	—	8	—	—	—	—	8
2	—	38	33	3	17	—	91
3	18	22	70	33	6	—	149
4	—	8	22	31	3	—	64
5	—	6	14	3	203	—	226
6	—	—	—	—	11	—	11
Totals	18	82	139	70	240	—	549

TABLE X.

Organic Correlation between Whiteness and Coat-Colour in Offspring of Hybrids.

Coat-Colour.	Amount of Whiteness.						
	1	*2*	*3*	*4*	*5*	*6*	Totals
a	1	4	8	6	9	4	32
b	3	17	28	17	25	6	96
c	11	36	39	15	46	5	152
d	—	5	8	2	4	—	19
e	2	15	19	23	3	16	78
f	2	9	10	3	1	10	35
g	—	—	2	—	2	2	6
Totals	19	86	114	66	90	43	418

TABLE XI.

Organic Correlation between Coat-Colour and Waltzing in Offspring of Hybrids.

	A	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
Waltzing ...	20	3	20	30	6	8	9	1	97
Not	117	29	76	122	13	70	26	5	458
Totals	137	32	96	152	19	78	35	6	555

TABLE XII.

Organic Correlation between Amount of Whiteness in Coat and Waltzing in Offspring of Hybrids.

	A	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Waltzing ...	20	9	18	19	8	17	6	97
Not	117	10	68	95	58	73	37	458
Totals	137	19	86	114	66	90	43	555

TABLE XIII.

Organic Correlation between Pinkness of Eye and Whiteness of Coat in Offspring of Hybrids.

	A	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Pink-eyed ...	137	6	28	37	22	23	18	271
Not	—	13	58	77	44	67	25	284
Totals	137	19	86	114	66	90	43	555

TABLE XIV.

Organic Correlation between Pinkness of Eye and Coat-Colour in Offspring of Hybrids.

	A	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
Pink-eyed ...	137	19	80	—	—	—	35	—	271
Not	—	13	16	152	19	78	—	6	284
Totals	137	32	96	152	19	78	35	6	555

TABLE XV.

Organic Correlation between Pinkness of Eye and Waltzing in Offspring of Hybrids.

	A	Pink-eyed	Dark-eyed	Totals
Waltzing ...	20	36	41	97
Not	117	98	243	458
Totals	137	134	284	555

TABLE XVI.

Fraternal Correlation in Coat-Colour among Offspring of Hybrids.

First Brother.

Second Brother.	*A*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
A	262	45	141	297	29	153	50	10	987
a	45	50	29	54	4	15	7	1	205
b	141	29	174	185	26	111	32	11	709
c	297	54	185	308	37	185	70	14	1150
d	29	4	26	37	8	14	11	1	130
e	153	15	111	185	14	92	34	5	609
f	50	7	32	70	11	34	30	1	235
g	10	1	11	14	1	5	1	2	45
Totals	987	205	709	1150	130	609	235	45	4070

TABLE XVII.

Fraternal Correlation in Whiteness of Coat among Offspring of Hybrids.

First Brother.

Second Brother.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
A	262	38	163	190	97	154	83	987
1	38	10	42	29	11	3	6	139
2	163	42	204	126	59	29	15	638
3	190	29	126	220	134	60	34	793
4	97	11	59	134	108	59	29	497
5	154	3	29	60	59	246	115	666
6	83	6	15	34	29	115	68	350
Totals	987	139	638	793	497	666	350	4070

TABLE XVIII.

Fraternal Correlation in Eye-Colour among Offspring of Hybrids.

First Brother.

Second Brother.	*A*	Pink-eyed	Dark-eyed	Totals
A	262	200	525	987
Pink-eyed ...	200	250	522	972
Dark-eyed ...	525	522	1064	2111
Totals	987	972	2111	4070

TABLE XIX.

Fraternal Correlation in Waltzing among Offspring of Hybrids.

First Brother.

Second Brother.	Waltzing	Not	Totals
Waltzing ...	124	566	690
Not... ...	566	2814	3380
Totals	690	3380	4070

TABLE XX.

Correlation between Coat-Colour in the Offspring of Hybrids and Purity of Albino Grandparents.

Coat-Colour in Young.

Purity of Albino Grandparents.	*A*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
Both Pure-bred	80	16	46	89	8	50	14	2	305
One Cross-bred	30	15	29	39	4	19	15	1	152
Both Cross-bred	27	1	21	24	7	9	6	3	98
Totals	137	32	96	152	19	78	35	6	555

TABLE XXI.

Correlation between Whiteness in Coat among Offspring of Hybrids and Purity of Albino Grandparents.

Whiteness in Young.

Purity of Albino Grandparents.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Both Pure-bred	80	6	35	54	45	58	27	305
One Cross-bred	30	10	26	37	13	22	14	152
Both Cross-bred	27	3	25	23	8	10	2	98
Totals	137	19	86	114	66	90	43	555

TABLE XXII.

Correlation between Pinkness of Eye in Offspring of Hybrids and Purity of Albino Grandparents.

Eye-Colour in Young.

Purity of Albino Grandparents.	*A*	Pink-eyed	Dark-eyed	Totals
Both Pure-bred ...	80	65	160	305
One Cross-bred ...	30	48	74	152
Both Cross-bred ...	27	21	50	98
Totals	137	134	284	555

TABLE XXIII.

Correlation between Waltzing in Offspring of Hybrids and Purity of Albino Grandparents.

Waltzing in Young.

Purity of Albino Grandparents.	Waltzing	Normal	Totals
Both Pure-bred ...	45	260	305
One Cross-bred ...	30	122	152
Both Cross-bred ...	22	76	98
Totals	97	458	555

TABLE XXIV.

Correlation between Whiteness in Offspring of Hybrids and in Waltzing Grandparents.

Whiteness in Young.

Whiteness in Waltzing Grandparents.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
$\alpha+\alpha$	18	3	23	34	14	11	9	112
$\alpha+\beta$	40	7	22	33	22	23	8	155
$\beta+\beta$	25	2	12	12	3	17	10	81
Totals	83	12	57	79	39	51	27	348

TABLE XXV.

Correlation between Coat-Colour in Offspring of Hybrids and Whiteness in Waltzing Grandparents.

Coat-Colour in Young.

Whiteness in Waltzing Grandparents.	*A*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
$\alpha+\alpha$	18	9	32	28	6	9	10	—	112
$\alpha+\beta$	40	13	26	39	6	25	6	—	155
$\beta+\beta$	25	1	14	21	2	12	4	2	81
Totals	83	23	72	88	14	46	20	2	348

TABLE XXVI.

Correlation between Eye-Colour in Offspring of Hybrids and Whiteness in Waltzing Grandparents.

Eye-Colour in Young.

Whiteness in Waltzing Grandparents.	*A*	Pink-eyed	Dark-eyed	Totals
$\alpha+\alpha$	18	37	57	112
$\alpha+\beta$	40	35	80	155
$\beta+\beta$	25	18	38	81
Totals	83	90	175	348

TABLE XXVII.

Correlation between Waltzing in Offspring of Hybrids and Whiteness in Waltzing Grandparents.

Waltzing in Young.

Whiteness in Waltzing Grandparents.	Waltzing	Normal	Totals
$a+a$	24	88	112
$a+\beta$	24	131	155
$\beta+\beta$	11	70	81
Totals	59	289	348

TABLE XXVIII.

Correlation between Whiteness of Coat in Offspring of Hybrids and Whiteness of Coat in Hybrid Father.

Whiteness in Offspring.

Whiteness in Hybrid Father.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
1	6	1	11	—	—	—	—	18
2	18	7	27	23	5	2	—	82
3	39	8	27	43	20	—	2	139
4	17	1	6	24	22	—	—	70
5	55	2	14	23	19	88	39	240
Totals	135	19	85	113	66	90	41	549

TABLE XXIX.

Correlation between Colour of Coat in Offspring of Hybrids and Whiteness in Hybrid Father.

Coat-Colour in Young.

Whiteness in Hybrid Father.	*A*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
1	6	—	1	9	—	1	1	—	18
2	18	6	18	22	3	11	4	—	82
3	39	8	16	41	5	20	10	—	139
4	17	1	24	12	4	10	1	1	70
5	55	17	35	68	7	34	19	5	240
Totals	135	32	94	152	19	76	35	6	549

TABLE XXX.

Correlation between Pinkness of Eye in Offspring of Hybrids and Whiteness in Hybrid Father.

Whiteness in Father.

Eye-Colour in Young.	*1*	*2*	*3*	*4*	*5*	Totals
Albino ...	6	18	39	17	55	135
Pink-eyed	2	26	27	21	56	132
Dark-eyed	10	38	73	32	129	282
Totals	18	82	139	70	240	549

TABLE XXXI.

Correlation between Waltzing in Offspring of Hybrids and Whiteness in Hybrid Father.

Waltzing in Young.	Whiteness in Father. 1	2	3	4	5	Totals
Waltzing ...	3	19	26	9	39	96
Normal ...	15	63	113	61	201	453
Totals	18	82	139	70	240	549

TABLE XXXII.

Correlation between Coat-Colour in Offspring of Hybrids and Coat-Colour in Hybrid Father.

Coat-Colour in Father.	Coat-Colour in Young. A	a	b	c	d	e	f	g	Totals
a	9	9	4	2	—	—	—	—	24
c	121	22	87	141	18	73	29	6	497
d	5	1	5	9	1	4	4	—	29
e	2	—	—	—	—	1	2	—	5
Totals	137	32	96	152	19	78	35	6	555

TABLE XXXIII.

Correlation between Whiteness in Offspring of Hybrids and Coat-Colour in Hybrid Father.

Coat-Colour in Father.	Whiteness in Young. A	1	2	3	4	5	6	Totals
a	9	—	—	2	1	12	—	24
c	121	17	84	107	63	71	34	497
d	5	1	2	4	2	7	8	29
e	2	1	—	1	—	—	1	5
Totals	137	19	86	114	66	90	43	555

TABLE XXXIV.

Correlation between Eye-Colour in Offspring of Hybrids and Coat-Colour in Hybrid Father.

Eye-colour in Young.	Coat-Colour in Father. a	c	d	e	Totals
Albino ...	9	121	5	2	137
Pink-eyed ...	4	120	8	2	134
Dark-eyed ...	11	256	16	1	284
Totals	24	497	29	5	555

TABLE XXXV.

Correlation between Waltzing in Offspring of Hybrids and Coat-Colour in Hybrid Father.

Coat-Colour in Father.

Waltzing in Young.	*a*	*c*	*d*	*e*	Totals
Waltzing ...	1	89	6	1	97
Normal ...	23	408	23	4	458
Totals	24	497	29	5	555

TABLE XXXVI.

Correlation between Colour of Coat in Offspring of Hybrids and Whiteness in Hybrid Mother.

Coat-Colour in Young.

Whiteness in Mother.	*A*	*a*	*b*	*c*	*d*	*e*	*f*	*g*	Totals
1	—	1	2	2	2	—	1	—	8
2	21	5	13	25	4	11	12	—	91
3	46	5	23	42	7	18	7	1	149
4	9	1	18	20	2	13	1	—	64
5	55	20	37	62	4	30	13	5	226
6	4	—	1	1	—	4	1	—	11
Totals	135	32	94	152	19	76	35	6	549

TABLE XXXVII.

Correlation between Whiteness of Coat in Offspring of Hybrids and Whiteness of Coat in Hybrid Mother.

Whiteness in Young.

Whiteness in Mother.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
1	—	—	7	1	—	—	—	8
2	21	8	29	23	5	3	2	91
3	46	5	35	47	16	—	—	149
4	9	3	6	19	26	1	—	64
5	55	3	8	22	19	85	34	226
6	4	—	—	1	—	1	5	11
Totals	135	19	85	113	66	90	41	549

TABLE XXXVIII.

Correlation between Eye-Colour in Offspring of Hybrid and Whiteness in Hybrid Mother.

Whiteness in Mother.

Eye-Colour in Young.	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Albino ...	—	21	46	9	55	4	135
Pink-eyed ...	3	27	32	14	54	2	132
Dark-eyed ...	5	43	71	41	117	5	282
Totals	8	91	149	64	226	11	549

TABLE XXXIX.

Correlation between Waltzing in Offspring of Hybrids and Whiteness in Hybrid Mother.

Waltzing in Young.	Whiteness in Mother. 1	2	3	4	5	6	Totals
Waltzing ...	1	21	24	12	37	1	96
Normal ...	7	70	125	52	189	10	453
Totals	8	91	149	64	226	11	549

TABLE XL.

Correlation between Coat-Colour in Offspring of Hybrids and Coat-Colour in Hybrid Mother.

Coat-Colour in Mother.	Coat-Colour in Young. A	a	b	c	d	e	f	g	Totals
a	13	17	7	7	—	—	—	—	44
c	108	12	70	129	18	64	24	6	431
d	14	3	19	14	1	13	6	—	70
e	2	—	—	2	—	1	5	—	10
Totals	137	32	96	152	19	78	35	6	555

TABLE XLI.

Correlation between Whiteness of Coat in Offspring of Hybrids and Coat-Colour in Hybrid Mother.

Coat-Colour in Mother.	Whiteness in Young. A	1	2	3	4	5	6	Totals
a	13	1	2	5	5	16	2	44
c	108	12	74	91	55	63	28	431
d	14	5	9	16	6	10	10	70
e	2	1	1	2	—	1	3	10
Totals	139	19	86	114	66	90	43	555

TABLE XLII.

Correlation between Eye-Colour in Offspring of Hybrids and Coat-Colour in Hybrid Mother.

Eye-Colour in Young.	Coat-Colour in Mother. a	c	d	e	Totals
Albino ...	13	108	14	2	137
Pink-eyed ...	11	97	21	5	134
Dark-eyed ...	20	226	35	3	284
Totals	44	431	70	10	555

TABLE XLIII.

Correlation between Waltzing in Offspring of Hybrids and Coat-Colour in Hybrid Mother.

Coat-Colour in Mother.

Waltzing in Young.	*a*	*c*	*d*	*e*	Totals
Waltzing ...	5	74	15	3	97
Normal ...	39	357	55	7	458
Totals	44	431	70	10	555

TABLE XLIV.

Organic Correlation between Coat-Colour and Whiteness in Offspring of Hybrids and Albinos.

Coat-Colour.

Amount of Whiteness.	*A*	*a*	*b*	*c*	*d*	*e*	*g*	Totals
A	368	—	—	—	—	—	—	368
1	—	—	—	—	—	—	—	—
2	—	1	—	—	2	4	1	8
3	—	3	4	17	1	10	1	36
4	—	1	1	27	7	15	3	54
5	—	15	7	101	27	4	2	156
6	—	3	—	22	18	77	4	124
Totals	368	23	12	167	55	110	11	746

TABLE XLV.

Correlation between Whiteness of Coat in Offspring of Hybrids × Albinos and the Purity of their Albino Grandparents. (A = Pure-bred Albino; A × = Cross-bred Albino. All four Grandparents are indicated, each pair in one bracket.)

Coat-Colour in Young.

Degree of Purity in Grandparents.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
(*A* . *A*) (*A* . *W*) ...	193	—	2	19	22	81	68	385
(*A* . *A*) (*A* × . *W*)...	70	—	2	6	12	39	21	150
(*A* . *A* ×) (*A* . *W*) ...	31	—	1	5	6	13	12	68
(*A* . *A* ×) (*A* × . *W*)...	18	—	2	6	3	5	2	36
(*A* × . *A* ×) (*A* . *W*) ...	17	—	—	—	1	8	8	34
(*A* × . *A* ×) (*A* × . *W*)...	27	—	1	—	4	7	9	48
Totals ...	356	—	8	36	48	153	120	721

TABLE XLVI.

Correlation between Coat-Colour in Offspring of Hybrid × Albino and the Purity of their Albino Grandparents. (The Grandparents indicated as in the preceding Table.)

Coat-Colour in Young.

Degree of Purity in Grandparents.	*A*	*a*	*b*	*c*	*d*	*e*	*g*	Totals
(*A*.*A*) (*A*.*W*) ...	193	17	6	75	32	58	4	385
(*A*.*A*) (*A*×.*W*) ...	70	1	2	45	13	15	4	150
(*A*.*A*×) (*A*.*W*) ...	31	1	1	15	3	17	—	68
(*A*.*A*×) (*A*×.*W*) ...	18	—	3	5	1	7	2	36
(*A*×.*A*×) (*A*.*W*) ...	17	3	—	8	2	3	1	34
(*A*×.*A*×) (*A*×.*W*) ...	27	1	—	14	1	5	—	48
Totals ...	356	23	12	162	52	105	11	721

TABLE XLVII.

Correlation between Whiteness in Offspring of Hybrid × Albino and Whiteness in Waltzing Grandparent.

Whiteness in Young.

Whiteness in Waltzing Grandparent.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
α	168	—	3	10	21	78	57	337
β	108	—	2	10	20	55	31	226
Totals	276	—	5	20	41	133	88	563

TABLE XLVIII.

Correlation between Coat-Colour in Offspring of Hybrid × Albino and Whiteness in Waltzing Grandparent.

Coat-Colour in Young.

Whiteness in Waltzing Grandparent.	*A*	*a*	*b*	*c*	*d*	*e*	*g*	Totals
α	168	12	2	74	34	40	7	337
β	108	8	5	52	12	37	4	226
Totals	276	20	7	126	46	77	11	563

TABLE XLIX.

Correlation between Coat-Colour in Offspring of Hybrid × Albino and Purity of Albino Parent.

Coat-Colour in Young.

Purity of Albino Parent.	*A*	*a*	*b*	*c*	*d*	*e*	*g*	Totals
Pure-bred ...	319	19	12	142	48	91	10	641
Cross-bred ...	49	4	—	25	7	19	1	105
Totals	368	23	12	167	55	110	11	746

TABLE L.

Whiteness of Coat in Offspring of Hybrid × Albino and Purity of Albino Parent.

Whiteness in Young.

Purity of Albino Parent.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
Pure-bred ...	319	—	5	35	42	138	102	641
Cross-bred ...	49	—	3	1	12	18	22	105
Totals	368	—	8	36	54	156	124	746

TABLE LI.

Whiteness in Offspring of Hybrid × Albino and Whiteness in Hybrid Parent.

Whiteness in Young.

Whiteness in Hybrid Parent.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
2	61	—	4	9	8	20	16	118
3	77	—	1	7	15	24	28	152
4	23	—	1	4	2	6	3	39
5	207	—	2	16	29	106	77	437
Totals	368	—	8	36	54	156	124	746

TABLE LII.

Coat-Colour in Offspring of Hybrid × Albino and Whiteness in Hybrid Parent.

Coat-Colour in Young.

Whiteness in Hybrid Parent.	*A*	*a*	*b*	*c*	*d*	*e*	*g*	Totals
2	61	2	1	31	6	15	2	118
3	77	—	3	30	13	26	3	152
4	23	—	3	8	—	4	1	39
5	207	21	5	98	36	65	5	437
Totals	368	23	12	167	55	110	11	746

TABLE LIII.

Whiteness in Offspring of Hybrid × Albino and Coat-Colour in Hybrid Parent.

Whiteness in Young.

Coat-Colour in Mother.	*A*	*1*	*2*	*3*	*4*	*5*	*6*	Totals
a	28	—	—	—	3	40	3	74
c	297	—	8	32	46	100	108	591
d	39	—	—	4	4	14	11	72
e	4	—	—	—	1	2	2	9
Totals	368	—	8	36	54	156	124	746

TABLE LIV.

Coat-Colour in Offspring of Hybrid × Albino and in Hybrid Parent.

Coat-Colour in Mother.	Coat-Colour in Young.							
	A	*a*	*b*	*c*	*d*	*e*	*g*	Totals
a	28	18	2	25	1	—	—	74
c	297	5	9	131	43	97	9	591
d	39	—	1	11	11	8	2	72
e	4	—	—	—	—	5	—	9
Totals	368	23	12	167	55	110	11	746

APPENDIX III.

Note on the Absence of anything like Telegony in the Albino Mice used.

It is well known that albino mice, paired together, breed true in coat-colour and in eye-colour. The following table, in which the notation is the same as that used in the previous part of the present paper, records a fairly long series of cases in which a female albino mouse was mated, first with a coloured mouse of some kind, and afterwards with an albino. The table shows clearly enough that in no single case did an albino doe, which had previously produced coloured young by a coloured buck, fail to produce a truly albino litter when subsequently mated with an albino buck.

I. First Non-Albino Mating		II. Albino Mating	I. First Non-Albino Mating		II. Albino Mating
Type of Buck	Offspring	Offspring	Type of Buck	Offspring	Offspring
Japanese Waltzing mouse	2*c*, 2*d*, 2*d*, 3*d*, 5*d*, 5*d*	4 alb.	"Extracted Dominant" 3*a* (*pw*) ...	5*a*, 5*c*, 5*c*, 4*c* & 4 alb.	8 alb.
Waltzer	2*c*, 2*c*, 2*c*, 2*c*, 2*c*, 2*d*	3 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	6 alb.
Waltzer	2*c*, 2*d*, 3*d*, 3*d*, 4*d*, 4*d*	(i) 1 alb. (ii) 5 alb.	"Extracted Dominant" 3*a* (*pw*) ...	5*bc* & 3 alb.	7 alb.
Waltzer	3*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*	4 alb.	Waltzer	2*c*, 2*c*, 3*c*, 5*c*, 5*c*, 5*c*, 5*c*	8 alb.
Waltzer	3*c*, 4*c*, 4*c*, 4*c*, 4*c*, 4*c*, 4*c*	2 alb.	Waltzer	2*c*, 2*c*, 2*c*, 2*c*, 3*c*	5 alb.
Waltzer	1*c*, 2*c*, 2*c*, 3*c*, 4*c*	3 alb.	Waltzer	6*c*, 5*d*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	1 alb.
Waltzer	4*c*, 4*c*, 4*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7 alb.	Waltzer	3*c*, 3*c*, 3*c*, 2*c*, 2*c*, 2*c*	8 alb.
Normal chocolate mouse	6*c*, 6*c*, 6*c*, 6*g*, 6*g*	6 alb.	Waltzer	5*c*, 5*c*, 2*c*, 2*c*, 2*c*, 3*c*	6 alb.
Hybrid (5*c*) ...	5*c*, 5*d*, 6*c* & 2 alb.	8 alb.	Waltzer	3*c*, 3*b* (*p*)	3 alb.
Hybrid (5*c*) ...	6*d*, 6*d* & 2 alb.	2 alb.	Black-eyed Fawn	6*a*, 6*a*, 6*a*, 6*c*, 6*g*	8 alb.
Hybrid (5*c*) ...	6*d*, 6*d*, 6*d*, 6*c* & 1 alb.	3 alb.	Hybrid (5*c*) ...	5*c*(very pale), 5*c*, 6*g* & 2 alb.	5 alb.
Waltzer	All still-born	5 alb.	Hybrid (2*c*) ...	5*c*, 5*d*, 5*c* & 5 alb.	8 alb.
Hybrid (3*c*) ...	4*c*, 4*d*, 4*d*, 5*d* & 5 alb.	5 alb.	Hybrid (3*c*) ...	6*c*, 6*c*, 6*c*, 5*d*, 6*d* & 3 alb.	7 alb.
Hybrid (3*c*) ...	6*c* & 5 alb.	3 alb.	Black mouse ...	6*c* & 1 alb.	4 alb.
Hybrid (3*c*) ...	4*d*, 4*d*, 4*d* & 3 alb.	4 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 4*c*	7 alb.
Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 2*c*, 3*c*, 3*c*, 3*c*	5 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*	4 alb.
Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 2*c*, 3*c*, 4*c*	5 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7 alb.
Hybrid (2*c*) ...	6*c*, 6*c* & 3 alb.	1 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7 alb.
Black mouse ...	6*c*, 3*c* & 6 alb.	6 alb.	Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*, 5*c*	7 alb.
Waltzer	3*c*, 5*c*, 5*c*, 5*c*	2 alb.	Hybrid (3*c*) ...	5*c* & 1 alb.	7 alb.
Waltzer	5*c*, 5*c*, 5*c*, 2*c*, 2*c*, 2*c*	6 alb.	Waltzer	3*c*, 3*c* (*w*), 3*c* (*w*), 4*c*	8 alb.
"Extracted Dominant" 3*a* (*pw*) ...	5*a* & 2 alb.	8 alb.	Waltzer	5*c*, 3*c*, 3*c*, 3*c*, 3*c*	7 alb.
" ...	5*b*, 5*b*, 5*b*, 5*b*, 6*b*, 6*b* & 1 alb.	9 alb.	Waltzer	5*c*, 5*c*, 4*c*, 3*c*, 3*c*	7 alb.
			Waltzer	5*c*, 5*c*, 5*c*, 5*c*, 5*c*	8 alb.

GRADUATION OF A SICKNESS TABLE BY MAKEHAM'S HYPOTHESIS.

By JOHN SPENCER, *Actuary.*

Some time ago an examination of certain sickness tables suggested to me the possibility that for a considerable period of life a law analogous to the well-known law of Mortality of Makeham might be found to apply to Sickness, i.e., that the force of sickness at any age x or the proportion of persons sick out of the number living at the precise moment of attaining that age might admit of being expressed in the form $A + Bc^x$, and I made various experiments to test the theory. Since that time Mr W. Palin Elderton, who had also investigated the same point, has referred to the matter in *Biometrika* *, and in a paper read in September, 1903, before the International Congress of Actuaries in New York, and has given the values of $\log_{10} c$ deduced by him from various sickness experiences.

The idea of a mathematical law governing Sickness is by no means a novel one. In a paper submitted to the International Statistical Congress in 1860 † Gompertz suggested that for ages between 20 and 60, S_x, the rate of sickness at age x or the average number of weeks' sickness experienced during a year by persons of that age, might be written kg^{c^x}, from which would follow the relation

$$\log S_x = A_1 + B_1 c_1^x.$$

Again Makeham himself dealt with the subject in 1871 when he read a paper "On the laws of Sickness and Invalidism" before the Institute of Actuaries ‡. He concluded from his investigation of data then available that the quantity h_x consisting of the number of healthy, as distinct from sick, persons aged x out of l_x the number living on the usual form of mortality table, was a function of the form $ks^x g^{c^x}$, this being the shape assumed by the l_x column when Makeham's law of mortality applies. This theory of Makeham's regarding sickness possesses one or two advantages of some importance and it differs from the other hypotheses

* Vol. II. p. 504.

† Reprinted in *Journal of Institute of Actuaries*, Vol. XVI. p. 329. See also *Phil. Trans.* Vol. 152, p. 554.

‡ *J. I. A.* Vol. XVI. p. 408.

mentioned above in introducing the element of mortality. It gives $\left(1 - \frac{h_x}{l_x}\right)$ as the force of sickness at age x and leads to the result

$$\operatorname{colog}\left(\frac{h_{x+1}}{h_x}\right) = A_2 + B_2 c_2^x.$$

The material which formed the subject of my investigation is embodied in Mr A. W. Watson's important work on the Sickness and Mortality Experience of the Manchester Unity Friendly Society for the five years 1893–1897*. The magnitude of the data there dealt with—embracing as they do no fewer than 39,000 deaths in the quinquennium and the record of upwards of 7,000,000 weeks of sickness—and the care and accuracy with which the observations have been analyzed and tabulated entitle the results disclosed in the volume to a degree of authority to which no previous investigation of the same character can lay claim. Mr Watson's exhaustive enquiry led him to deal with his voluminous data upon novel lines, and we find tabulated the mortality experience of various sections of the Society grouped according to geographical situation, while as regards Sickness the element of occupation was the factor which determined the classification, the experience of each of the four following groups of trades being published:—

(*a*) Agricultural and General (Normal) occupations;

(*b*) Building Trades, Railway, Seafaring and Outdoor Labouring Occupations;

(*c*) Quarry Workers, Iron, Steel and Chemical Workers, &c.; and

(*d*) Mining Occupations.

The first of these groups is the one to which the following notes relate. It consists, broadly speaking, of persons engaged in occupations involving no special hazard, and comprises nearly 80 per cent. of the membership of the Society. These lives were during the quinquennium exposed to risk of sickness for 2,352,099 years, while they experienced in the aggregate 5,289,586 weeks of sickness in the period.

Throughout the investigation in determining the quantity "Exposed to Risk of Sickness" it was assumed, with a particular object in view, that lives dying in any calendar year during the period of observation were at risk until the middle of the year. In consequence of this assumption the ungraduated rates of sickness which were obtained by dividing the number of weeks' sickness experienced during the year by persons of a particular age by the quantity "Exposed to Risk" at that age differ from the rates of sickness ordinarily tabulated. They represent rather what might be termed central rates of sickness, or, approximately speaking, values of the force of sickness at the middle of the year. The latter consideration enables us on the hypothesis which I am discussing to write the tabulated rate of sickness in the form $A + Bc^{x+\frac{1}{2}}$ and suggests a simple method of deducing the values of the constants. Having for reasons which will appear presently come to the

* London: C. and E. Layton. 1903.

conclusion that a Makeham curve* would not apply throughout the whole of the table, i.e., from age 16 to 100, I ultimately divided the data for ages 19—78 into three groups consisting respectively of ages 19—38, 39—58, and 59—78. The ungraduated rates in each of these groups were summed, the resulting totals being

$$20A + Bc^{19\frac{1}{2}} \cdot \frac{c^{20}-1}{c-1},$$

$$20A + Bc^{39\frac{1}{2}} \cdot \frac{c^{20}-1}{c-1},$$

and

$$20A + Bc^{59\frac{1}{2}} \cdot \frac{c^{20}-1}{c-1}.$$

Differencing these, we have an obvious means of deducing the value of $\log c^{20}$, and hence the values of the other constants.

The following are the constants resulting from this grouping†:—

$A = \cdot 747127.$ $\quad B = \cdot 00680912.$ $\quad \log_{10} c = \cdot 0462118.$

The values of A and B are here given in weeks, and would have to be divided by $52\frac{1}{6}$ to furnish the corresponding yearly values.

At the outset, in considering to what extent of the table a Makeham curve could be fitted, one or two points presented themselves which made it clear that the hypothesis would not apply for the whole of life. At the beginning of the table the evidence points in my view to a fall for a few years in the rate of sickness, a feature which is not singular to this section of the Society's experience but is reflected in the observations relating to other occupation groups. Again towards the later years of life, i.e., from about age 75, there is an unmistakable decline in the ratio at which the rate of sickness increases and a tendency towards a constant rate of sickness of about 40 weeks per annum as will be seen from the following average unadjusted rates:—

Age group	Average rate of sickness in weeks
68—72	13·78
73—77	21·24
78—82	29·02
83—87	34·72
88—92	37·25
93—97	39·73

Considerable weight may I think be attached to these quinquennial rates since they are based upon a relatively large number of observations, one of the distinctive

* In speaking here, and in what follows, of a Makeham curve or graduation I refer to the hypothesis formulated at the beginning of this paper and not to Makeham's own theory of sickness.

† [Mr W. Palin Elderton has at my suggestion worked out $\log_{10} c$ by the method indicated by him in *Biometrika*, Vol. II. p. 503. To fit Mr Spencer's range from 19 to 78, he took the origin of his exposed to risk normal curve at 42·5 years, and he found $\log_{10} c = \cdot 046{,}0043$. Considering the complete difference of method, Mr Elderton's result closely confirms Mr Spencer's value. K. P.]

features of the new Manchester Unity Experience being the extent of the data at the old ages.

It is of course clear that neither the decline in the rate of sickness from age 16 to about age 20 nor the bend in the curve exhibited at the other end of the table could be reproduced by a Makeham graduation without the aid of supplementary curves, and as for the moment I was concerned not so much with a graduation of the whole table as with an attempt to fit an $A + Bc^x$ curve to as great an extent of the table as possible I decided to confine my attention to the facts for ages 19—80. I made various groupings all of which gave a value of $\log_{10} c$ approximating to ·046. Eventually however I chose the data for ages 19—78 for closer investigation and from these deduced the constants given above.

The following table shows rates of sickness based on these constants. For purposes of comparison the ungraduated rates are also given, together with the adjusted rates published in Mr Watson's volume. These latter it may be remarked were obtained from the rough rates by the application of a 15 term summation formula.

Central Rates of Sickness.

Age	Rates on Makeham Hypothesis	Ungraduated Rates	Official Adjusted Rates	Age	Rates on Makeham Hypothesis	Ungraduated Rates	Official Adjusted Rates
20	·807	·823	·815	*50*	2·215	2·146	2·177
21	·814	·808	·810	*51*	2·380	2·358	2·347
22	·822	·810	·814	*52*	2·564	2·550	2·539
23	·830	·824	·823	*53*	2·767	2·764	2·750
24	·839	·837	·834	*54*	2·994	2·958	2·978
25	·850	·853	·845	*55*	3·247	3·240	3·221
26	·861	·851	·856	*56*	3·527	3·496	3·481
27	·874	·881	·867	*57*	3·839	3·773	3·759
28	·888	·876	·881	*58*	4·186	4·096	4·065
29	·904	·890	·898	*59*	4·573	4·398	4·413
30	·922	·917	·918	*60*	5·002	4·799	4·821
31	·942	·952	·941	*61*	5·480	5·320	5·310
32	·963	·959	·966	*62*	6·013	5·827	5·893
33	·988	1·000	·991	*63*	6·602	6·523	6·570
34	1·015	1·009	1·020	*64*	7·260	7·394	7·335
35	1·045	1·063	1·052	*65*	7·991	8·190	8·183
36	1·078	1·086	1·091	*66*	8·804	9·290	9·117
37	1·115	1·127	1·136	*67*	9·709	10·035	10·153
38	1·157	1·176	1·190	*68*	10·715	11·136	11·302
39	1·203	1·257	1·249	*69*	11·834	12·472	12·559
40	1·254	1·315	1·313	*70*	13·079	14·092	13·913
41	1·311	1·392	1·378	*71*	14·463	15·384	15·346
42	1·374	1·437	1·443	*72*	16·003	16·952	16·846
43	1·444	1·510	1·508	*73*	17·716	18·202	18·409
44	1·522	1·582	1·574	*74*	19·621	19·951	20·036
45	1·610	1·632	1·643	*75*	21·740	21·619	21·713
46	1·706	1·726	1·717	*76*	24·098	23·641	23·397
47	1·814	1·813	1·804	*77*	26·719	25·055	25·061
48	1·934	1·877	1·907	*78*	29·635	26·776	26·688
49	2·067	2·037	2·030	*79*	32·878	28·050	28·265

It will be seen that on the whole my graduated rates run very closely to the unadjusted values until about age 55. Above that age the agreement is not invariably so marked and from age 75, as might have been anticipated, the two series diverge rapidly. Obviously the graduation fails to fit the raw data above age 80 and, apart from this circumstance, the fact that after a certain point the value of the force of sickness would exceed unity, or $52\frac{1}{6}$ weeks per annum, forms a theoretical objection to the hypothesis as applied to the final ages in the table. Makeham's own theory of sickness avoids the latter difficulty, though, after investigation, I conclude that it fails satisfactorily to represent the sickness after age 80 in the table under examination.

To show the extent to which the graduated rates of sickness reproduce the actual sickness experienced when multiplied at each age by the number "Exposed to Risk" I now give the following comparison:—

Age Group	Expected weeks of Sickness	Actual weeks of Sickness	Excess of Expected over Actual	Ratio per cent. of Excess to Actual
16—29	659,739	658,029	+ 1,710	+ ·3
30—39	645,877	652,620	− 6,743	−1·0
40—49	700,681	715,141	−14,460	−2·0
50—59	938,759	924,705	+14,054	+1·5
60—69	1,035,598	1,046,278	−10,680	−1·0
70—79	1,009,216	1,005,771	+ 3,445	+ ·3
Totals	4,989,870	5,002,544	−12,674	− ·25

As a final test of the graduation I append the values at 3% interest of a benefit of £1 per week during sickness throughout life as compared with those published officially, the mortality assumed in each case being that exhibited by the members of the Society in Non-Manufacturing Districts, designated Area 1 in Mr Watson's volume. For the purpose of calculating the former values it was necessary to deal with the sickness above age 79 and I made the convenient assumption of a constant rate of sickness from age 80 onwards of 34·634 weeks per annum, this being the value of $A + Bc^{80}$ given by my constants. The average rate of sickness at these ages according to the observations is only 32·724 weeks per annum, but the adoption of the higher value not only avoids the assumption of a diminishing rate of sickness but introduces a factor which compensates for the deficiency of Expected Sickness up to age 79. Strictly speaking perhaps I should have worked out monetary values from the unadjusted data for comparison with those now calculated, but since the graduation upon which the official values are based is in effect a smoothing-down process which reproduces the prominent characteristics of the rough data, the published values may confidently be employed as a standard of comparison.

Values at 3 per cent. of a Benefit of £1 per week during Sickness throughout life.

Age	On Makeham Hypothesis	As published
	£	£
20	57·44	57·39
25	63·26	63·22
30	70·25	70·24
35	78·35	78·32
40	87·74	87·58
45	98·54	98·01
50	110·50	109·88
55	123·84	123·21
60	137·67	137·34

PRELIMINARY NOTE ON THE PROTECTIVE VALUE OF COLOUR IN MANTIS RELIGIOSA.

By A. P. di CESNOLA, Queen's College, Oxford.

During last summer vacation, at Professor Weldon's suggestion, I undertook a small series of experiments upon protective coloration from a statistical standpoint. The form chosen was *Mantis religiosa*, which occurs in Italy (where the experiments were made) in two forms, a green and a brown. It is interesting to notice that the green form is always found upon green grass, the brown form upon grass burnt by the sun. The green form is characterised by its more sluggish behaviour; the brown form is more active.

For the purpose of this experiment I collected 110 specimens of *Mantis religiosa*, 45 green and 65 brown. Each individual was tied by a silk thread about six inches long to a plant, the thread being tied round the animal's thorax, and each *Mantis* being attached to a separate plant.

The individuals were divided into four groups, two green and two brown; of the green individuals, 20 were tied to green plants in a place covered with green herbage, the remaining 25 being tied to brown plants in a place where the herbage generally was burnt and brown. In the same way, 20 of the brown individuals were tied to brown plants in a burnt-up spot, and 45 were tied to green plants on a green piece of ground. The whole series was tied in this way and exposed on August 15th, and observations were made daily during seventeen days.

The death-rate during the period of observation may be gathered from the diagrams Figs. 1 and 2, and it will be seen that during the whole period the 20 green individuals exposed among green grass, and the 20 brown individuals exposed among brown grass were untouched by enemies, so that all of them survived.

Of the 25 green individuals exposed upon brown grass, the last was killed by August 25th, eleven days after the commencement of the experiment. Of the 45 brown individuals, exposed upon green grass, ten were left on September 1st. On the evening of September 1st a gale occurred, which ultimately swept away the remaining individuals, and brought the experiment to a conclusion.

Of the individuals which died, nearly all were killed by birds; but of the 25 green individuals exposed on brown grass, five were killed by ants.

It will of course be necessary to repeat the experiments upon a larger scale; but they seem to show in a fairly convincing manner the value of protective coloration.

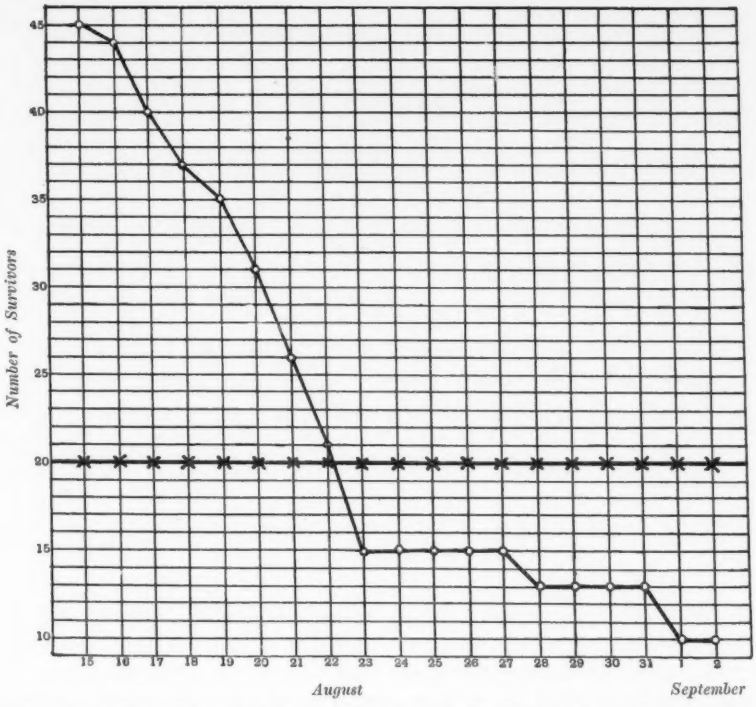


FIG. 1. Record of observations of 65 brown individuals of *Mantis religiosa*, 45 exposed on green and 20 on brown herbage. The circles show the number of individuals on the green herbage alive on each day of the experiment, the crosses the number of individuals alive on brown herbage.

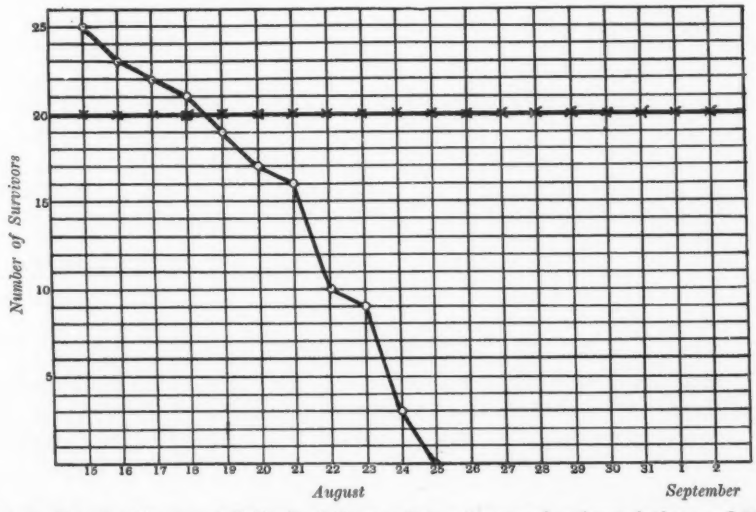


FIG. 2. Record of 45 green individuals of *Mantis religiosa*, 25 exposed on brown herbage, and 20 on green. The circles show the number of those on brown herbage surviving on each day of the experiment, the crosses show the number of survivors on green herbage.

MEASUREMENTS OF ONE HUNDRED AND THIRTY CRIMINALS.

By G. B. GRIFFITHS, M.R.C.S., L.R.C.P., Dy. Medical Officer, H.M. Prison Service.

WITH INTRODUCTORY NOTE

By H. B. DONKIN, M.D., F.R.C.P., One of H.M. Prison Commissioners and Directors of Convict Prisons.

THESE Tables are intended only as an indication of the method of certain anthropometrical observations which are now being carried out by the good-will of the Medical Officers of the English Convict Prisons. No conclusion from such a small number of cases is, of course, to be attempted. The whole scheme, of which these notes are but a sample instalment, has for its object the collection of large numbers of observations, anthropometrical and otherwise, on criminals undergoing sentences of three years and upwards, without any selection whatever. It is hoped that in the course of time such results will be attained, as may possibly throw some light on criminological questions which have been raised, and sometimes prematurely solved, by various writers.

The present preliminary observations were made by Dr Griffiths at Parkhurst Prison according to forms decided on in consultation with Dr Smalley, the Medical Inspector of Prisons, and myself. It is to be noted that they differ from those which will be made in future in that a certain selection of cases has been made, the larger scheme not having been completely formulated at the time.

H. B. DONKIN.

Method of Measurement.

The person to be measured is seated and looking directly to his front, the head being in the "Horizontal Position," i.e. so that a pen placed clerkwise over the ear will join the external angle of the eye and the topmost junction of the external ear and head.

Next, a steel tape, weighted at either end, is placed on the head at the External Occipital Protuberance and carried forward along the Vertex and middle of the forehead to the chin. The weight will keep this tape in position and a similar weighted tape is placed coronally from one External Auditory Meatus to the other over the Vertex. Where these steel tapes cut each other on the Vertex it is agreed for the purposes of these investigations to call the Bregma.

The Antero-posterior Curve is the first measurement taken by means of a steel measuring tape joining the Nasion and External Occipital Protuberance. This curve will be cut into an Anterior and a Posterior segment by the tape stretching from one External Auditory Meatus to the other.

Next, with the crossed tapes still in position, the Horizontal Curve is taken by passing the measuring tape round the head from the Ophryon along the right side, over the External Occipital Protuberance and forward along the left side to the Ophryon again. The crossed tapes will cut this Circumference into four moieties:

(i) Right Anterior.

(ii) Right Posterior.

(iii) Left Posterior.

(iv) Left Anterior.

In taking this measurement, (1) the whole Circumference is noted, then (2) the distance from the Ophryon to the Right Auricular line, i.e. the tape joining the External Meatus; next, the number on the tape at the External Occipital Protuberance where it cuts the Anterior Posterior one; next, the number on the Left Auricular line.

Supposing (1) to be 15 cm., that is the measurement of the Right Anterior Segment, and (2) that the tape marks 30 cm. over the External Occipital Protuberance, $30 - 15 = 15$, which is the measurement of the Right Posterior Segment. The two remaining segments are similarly got by subtraction.

The Cranial Height is taken:—a vertical standard with a horizontal arm at right angles which will slide up and down is placed by the subject's side with the horizontal arm pointing coronally across the head and just touching the Bregma. One limb of the callipers is placed in the middle of the External Auditory Meatus, and the other so as just to touch the lower surface of the horizontal arm. The vertical distance is the Vertical Height.

Cranial Length is the direct distance from the Glabella to the External Occipital Protuberance.

Cranial Breadth is the greatest Bi-parietal Diameter.

The Face Length is taken as the distance from just below the middle point of the chin to the Nasion.

The Face Breadth is the greatest Bi-Zygomatic Diameter.

"Facial Symmetry" is not a measurement but is judged by carrying the steel tape from the External Occipital Protuberance forward to the middle point of the Chin and noticing which side is greater.

"Nasal Deflection" is judged in a similar manner.

The Curve of the Forehead is taken:—the limbs of the callipers are fixed at 10 cm. apart, and one limb placed on the Nasion and the other on the Vertex in the middle line. The curve between these two points is measured with the tape and is the Forehead Curve.

The Auricular-Alveolar, Auricular-Nasal, Auricular-Occipital, and Auricular-Mental Diameters are taken with the callipers, and are the distances between the Middle of the External Auditory Meatus and the Alveolar Point, the Nasion, the External Occipital Protuberance, and the Chin respectively.

The Chin and Occipital Projections are taken by placing the standard behind the subject with the horizontal arm pointing forward. A weighted tape is hung from the horizontal arm and hangs plumb, just touching the External Occipital Protuberance or Chin; and the distance taken by callipers between the External Auditory Meatus and a steel bar at right angles to this plumb-line is the Projection required.

The head measurements are all expressed in millimetres.

The Horizontal Circumference is divided into four segments to facilitate the comparison of the Anterior and Posterior and Right and Left Segments.

The following terms and abbreviations are used in describing the shape of the Nose:—

Rect. = Rectilinear, Concave, Convex, Hump, Und. = Undulating, refer to shape from Nasion to tip; Hor. = Horizontal, Elev. = Elevated, Desc. = Descending, refer to angle from tip to junction with upper lip.

The present investigations have been made on 100 "Ordinary" and 30 Lunatic Criminals.

Reg. and Crime	Cranium															Face					Fo
	Antero-posterior Curve	Ant. Segment	Post. Seg.	Horizontal Circum.	Right Ant.	R. Post.	L. Post.	L. Ant.	Ant. Seg.	Post. Seg.	Cranial Height	Cranial Length	Cranial Breadth	Cephalic Index	Vertical Index	Length	Breadth	Facial Index	Symmetry	Eye-brows. C=concurrent N=not concurrent	Curve
ZH 127 Wounding with intent to murder	315	180	135	558	145	140	140	133	278	280	121	198	145	73·2	60·7	123	133	108·1	=	C	110
N 532 ,,	300	160	140	545	130	150	135	130	260	285	140	192	153	79·6	72·9	120	138	115·0	=	C	112
WGN 964 ,,	310	165	145	560	145	145	145	125	270	290	139	197	155	77·8	70·5	129	133	103·1	L+	C	110
L 623 ,,	300	155	145	563	130	150	160	123	253	310	138	190	155	81·5	72·6	129	136	105·4	L+	C	110
U 327 ,,	315	160	155	565	120	155	145	145	265	300	136	193	155	80·3	70·4	125	143	114·4	=	C	105
U 88 ,,	305	150	155	550	140	150	130	130	270	280	127	198	145	73·2	64·1	138	138	100·0	L+	C	115
V 356 ,,	300	160	140	555	130	140	150	135	265	290	138	190	150	78·9	72·6	129	141	109·3	L+	N	110
KP 260 ,,	350	170	180	548	140	100	175	133	273	275	136	194	146	75·2	70·1	121	132	109·0	=	C	110
X 53 ,,	315	160	155	555	130	140	155	130	260	295	135	192	155	80·7	70·3	118	139	117·7	L+	C	108
V 357 ,,	305	160	145	540	130	130	150	130	260	280	112	185	143	77·8	60·2	120	135	112·5	L+	C	112
W 433 ,,	305	165	140	550	140	145	135	130	270	280	114	194	144	74·2	58·7	134	136	101·4	=	N	105
X 176 ,,	308	180	128	560	130	140	140	150	280	280	137	198	155	78·2	69·1	134	137	102·7	=	C	104
Y 428 ,,	305	165	140	545	130	140	135	140	270	275	118	187	156	83·4	63·1	127	143	112·5	=	C	110
Z 134 ,,	325	165	160	590	140	150	160	140	280	310	135	205	160	78·0	65·8	130	135	103·8	=	C	110
Z 426 ,,	308	160	148	565	135	135	150	145	280	285	132	195	155	79·4	67·6	132	142	107·5	L+	C	103
Z 225 ,,	315	180	135	585	145	145	150	145	290	295	119	200	160	80·0	59·5	120	142	118·3	L+	C	115
a 74 ,,	320	180	140	580	135	160	140	145	280	300	143	199	161	80·9	71·8	130	140	107·7	=	C	104
a 45 ,,	300	160	140	560	125	165	135	135	260	300	121	195	150	76·8	62·0	126	133	105·5	=	C	115
a 137 ,,	280	170	110	535	135	145	130	125	260	275	130	180	155	86·1	72·2	114	143	125·5	=	C	105
a 370 ,,	315	168	147	545	140	145	135	125	265	280	135	194	153	78·4	71·6	124	137	110·4	L+	C	104
a 420 ,,	300	150	150	550	140	140	140	130	270	280	124	195	150	76·9	63·5	110	135	122·7	L+	N	110
b 120 ,,	292	165	127	530	130	135	130	135	265	265	129	174	157	90·2	74·1	124	145	117·9	=	C	112
b 467 ,,	320	160	160	545	140	130	135	140	280	265	124	190	147	77·3	65·2	117	155	115·3	L+	C	110
b 326 ,,	300	155	145	540	140	130	130	140	280	260	120	187	146	78·0	64·1	112	133	118·7	L+	C	115
b 343 ,,	350	190	160	580	160	135	145	140	300	280	136	197	149	75·6	69·0	136	130	95·5	R+	C	105
IX 472 ,,	350	180	150	575	150	135	145	145	295	280	137	200	160	80·0	68·5	130	146	112·3	L+	C	105
b 252 Forgery ...	340	165	175	595	135	160	170	130	265	330	140	210	163	77·6	66·6	127	136	107·0	R+	C	105
b 371 Embezzlement	294	160	134	555	125	145	140	145	270	285	137	195	150	76·9	70·2	120	138	115·0	=	C	110
a 433 Falsifying Accts	315	155	160	570	140	145	145	140	280	290	143	197	160	81·2	72·5	140	148	105·7	L+	N	110
b 372 Forgery ...	315	160	155	560	140	140	150	130	270	290	133	193	150	77·7	68·8	130	135	103·8	=	C	110
X 69 Theft	340	180	160	580	150	150	140	140	290	290	128	196	153	78·0	65·3	120	143	119·1	=	C	110
b 265 ,, ...	305	160	145	570	140	140	150	140	280	290	128	193	160	82·9	66·3	123	139	113·0	L+	N	110
b 378 Felony	320	170	150	545	135	145	130	135	270	275	135	185	140	75·6	72·9	135	136	100·7	L+	C	105
b 301 Fraud	330	165	165	580	130	160	150	140	270	310	142	196	160	81·6	72·4	132	147	111·3	=	C	110
a 339 Theft	300	165	135	545	140	130	150	125	265	280	130	186	148	79·5	69·8	127	130	102·3	L+	C	110
L 466 ,, ...	320	155	165	555	130	140	155	130	260	295	130	188	150	79·7	69·1	146	143	97·9	=	N	105
a 141 Receiving ...	350	195	155	610	160	145	185	120	280	330	136	210	162	77·1	64·7	135	150	111·1	L+	C	115
Na 48 Theft	310	165	145	540	130	130	150	130	260	280	128	190	153	80·5	67·3	125	135	108·0	L+	N	110
a 474 Burglary ...	345	190	155	570	130	150	150	140	270	300	138	203	152	74·8	67·9	127	140	110·2	L+	C	110
b 462 Theft	330	175	155	565	130	150	150	135	265	300	138	200	150	75·0	69·0	127	138	108·6	L+	C	110
b 349 Felony	320	160	160	560	140	140	140	140	280	280	139	192	144	75·0	72·3	132	135	102·2	L+	C	115
b 361 Theft	320	160	160	560	140	150	120	150	290	270	130	195	156	80·0	66·6	120	143	119·1	=	N	105
Wb 361 Theft ...	310	170	140	545	150	120	150	125	275	270	143	187	146	78·0	76·4	121	137	113·2	L+	C	105
TNS*b* 335 Theft ...	310	155	155	565	145	135	155	130	275	290	124	193	156	80·0	64·2	137	137	100·0	L+	C	105
b 357 Shop breaking...	320	180	140	550	140	150	110	150	290	260	130	194	149	76·8	67·0	112	134	119·6	L+	N	120
b 321 ,, ...	310	160	150	540	125	140	155	120	245	295	130	187	151	80·0	69·5	118	137	116·1	=	C	110
LKT*b* 242 Burglary ...	310	155	155	575	140	145	160	130	270	305	129	205	146	71·2	62·9	117	128	109·4	L+	C	115
DMQL 289 Theft ...	320	170	150	560	140	135	155	130	270	290	140	198	150	75·7	70·7	117	140	119·7	L+	C	110
b 320 Receiving ...	310	165	145	555	140	140	140	135	275	280	132	181	162	89·5	72·9	122	148	121·3	L+	C	105
LISY 1261 Theft ...	310	155	155	575	140	140	160	135	275	300	135	200	155	77·7	67·5	135	132	97·7	L+	C	110
P 140 Murder ...	320	165	155	563	140	145	145	133	273	290	157	194	156	80·4	80·9	134	136	101·4	=	C	105
Q 91 ,, ...	330	158	172	565	140	130	155	140	280	285	136	192	151	78·6	70·8	123	139	113·0	L+	C	110
Q 433 ,, ...	315	178	137	550	150	135	130	135	285	265	153	192	150	78·1	79·6	132	146	110·6	R+	N	110
R 126 ,, ...	320	167	153	550	145	135	130	140	285	265	139	189	147	77·7	73·5	112	129	115·1	R+	C*	110
R 388 ,, ...	305	158	147	530	130	120	150	130	260	270	142	187	145	77·5	75·8	114	133	116·6	R+	C	110
T 397 ,, ...	320	175	145	555	145	135	140	135	280	275	143	195	147	75·3	73·3	125	132	105·6	=	C	112
R 475 ,, ...	325	177	148	570	150	145	140	135	285	285	139	199	150	75·3	69·8	127	127	100·0	R+	C	105
T 403 ,, ...	310	175	135	560	155	140	145	120	275	285	134	193	154	79·7	69·4	121	137	113·2	R+	C	110
M 547 ,, ...	320	180	140	560	140	135	145	140	280	280	138	195	155	79·4	77·0	129	141	109·2	=	C	105
H 826 ,, ...	295	145	150	545	128	146	141	130	258	287	132	190	146	76·8	69·4	128	131	102·3	=	C	105
T 729 ,, ...	350	155	195	573	130	160	155	129	259	315	153	200	149	74·5	76·5	126	143	113·4	R+	C	110
M 145 ,, ...	320	155	165	558	145	135	146	132	277	281	138	193	155	80·3	71·5	122	126	103·2	=	N	104
M 744 ,, ...	315	155	160	543	138	142	135	128	266	277	140	193	147	75·6	72·5	123	123	100·0	=	N	102
N 399 ,, ...	320	160	160	565	145	150	145	125	270	295	140	200	153	76·5	70·0	129	121	93·7	=	N	110
N 63 ,, ...	305	155	150	565	140	155	145	125	265	300	147	190	154	81·0	77·3	117	127	108·5	=	C	110
Q 430 ,, ...	310	158	152	565	130	150	143	142	272	293	135	203	156	76·8	66·5	139	136	97·8	R+	C	110
U 548 ,, ...	345	185	160	583	150	150	145	138	288	295	142	205	152	74·1	69·2	126	142	112·6	R+	C	110
V 207 ,, ...	320	165	155	550	145	135	148	124	269	283	138	190	144	75·7	72·5	130	133	102·3	=	C	115
W 144 ,, ...	310	155	155	548	130	145	150	123	253	295	139	193	146	75·6	72·0	114	130	114·0	L+	C	115
V 384 ,, ...	345	172	173	550	145	135	135	135	280	270	122	197	138	70·0	61·9	120	123	102·5	=	C*	108
W 313 ,, ...	315	145	170	550	140	140	145	125	265	285	119	185	150	81·0	64·0	126	122	96·8	=	C	117
W 504 ,, ...	335	155	180	546	140	140	145	121	261	285	130	186	150	80·6	69·8	120	133	110·8	=	C	110

* Concurrence slight.

Eye-brows. C = concurrent N = not concurrent	Forehead: Curve	Forehead: High (H), Low (L), Receding (R), Med. (M), Broad (B)	Nose: General Shape	Nose: Deflection	Auriculo-Alveol. Diameter	Auriculo-Nasal Diameter	Gnathic Index	Auriculo-Occip. Diameter	Occipital Projection	Auriculo-Mental Diameter	Chin Projection	Ears: Length—Right	Ears: Length—Left	Ears: Lobule. A = adherent N = not adherent	General Observations
C	110	HB	Concave, Elev.	L	122	120	101·6	133	100	145	101	70	68	N	
C	112	LR	Concave, Elev.	L	109	110	99·0	134	107	134	90	70	68	A	
C	110	MR	Rect., Hor.	L	115	116	99·0	148	108	130	85	69	69	N	
C	110	HB	Hump, Hor.	L	110	114	96·4	140	118	133	91	59	60	N	
C	105	MB	Concave, Hor.	Nil	110	116	94·8	133	119	134	96	52	62	N	
C	115	MR	Convex, Elev.	L	118	124	95·1	135	125	143	106	65	66	A	
N	110	MB	Rect., Elev.	R	114	112	101·7	140	117	130	91	61	61	N	
C	110	MBR	Und., Elev.	R	117	115	101·7	141	102	130	91	66	65	A	Locomotor Ataxy
C	108	HB	Concave, Elev.	R	118	120	98·3	128	93	132	88	58	58	N	W. M.
C	112	HB	Hump, Hor.	R	119	116	102·5	124	113	131	103	55	56	A	W. M.
N	105	MB	Und., Hor.	R	117	118	99·1	134	110	133	90	68	68	N	
C	104	MB	Rect., Hor.	Nil	115	120	95·8	140	127	135	95	63	64	A	
C	110	HB	Und., Hor.	R	117	119	98·3	117	108	137	104	70	72	N	[tuberance
C	110	HSq.	Concave, Elev.	R	115	116	99·1	139	106	132	98	63	70	A	Depression above ext. occip. pro-
C	103	HB	Rect., Hor.	R	112	116	96·5	140	104	123	96	65	66	A	
C	115	HB	Und., Elev.	L	115	117	98·2	127	100	139	97	62	64	N	Depression above ext. occip. pro-
C	104	HB	Und., Rect.	Nil	121	120	100·8	134	98	134	93	61	66	N	[tuberance. German
C	115	MB	Rect., Elev.	L	110	113	97·3	124	112	130	89	53	56	N	
C	105	LB	Concave, Elev.	Nil	107	110	97·2	127	114	95	128	64	66	A	
C	104	LB	Rect., Hor.	R	115	113	101·7	142	85	127	88	62	62	A	
N	110	MB	Und., Desc.	R	113	113	100·0	133	99	135	100	63	68	A	
C	112	HB	Convex, Desc.	R	110	112	98·3	129	94	136	94	60	58	A	
C	110	MB	Und., Elev.	R	114	110	103·6	127	107	129	100	57	59	A	
C	115	MB	Rect., Hor.	R	113	114	99·1	120	117	120	106	49	55	A	
C	105	MB	Concave, Elev.	R	118	119	99·1	130	96	135	105	59	62	A	
C	105	MB	Concave, Hor.	L	119	123	96·7	135	96	135	113	63	63	N	
C	105	HB	Hump, Desc.	R	109	116	94·3	145	91	135	104	63	63	N	
C	110	MB	Rect., Elev.	R	108	120	90·0	130	102	120	92	56	57	A	Rudiment of Darwin's Tubercle
N	110	HB		R	117	122	95·9	138	107	138	107	62	62	N	
C	110	MB	Concave, Hor.	R	115	115	100·0	127	112	118	98	59	58	A	Rudiment of Darwin's Tubercle
C	110	MB	Hump, Rect.	L	116	118	98·3	118	128	140	109	62	58	A	Darwin's Tubercle
N	110	MB	Concave, Elev.	R	113	114	99·1	132	113	126	80	50	57	A	
C	105	LB	Hump, Rect.	R	121	119	101·6	120	115	135	105	63	63	A	
C	110	MB	Hump, Rect.	Nil	117	120	97·5	130	104	140	113	61	62	N	
C	110	MB	Concave, Elev.	R	110	110	100·0	120	105	120	94	55	59	A	
N	105	MB	Hump, Rect.	Nil	120	118	101·7	125	112	132	105	63	63	A	
C	115	HB	Concave, Elev.	R	124	124	100·0	134	113	140	95	65	70	A	
N	110	MB	Concave, Elev.	R	113	115	98·2	130	85	125	102	64	64	A	
C	110	LR	Und., Elev.	R	120	120	100·0	130	110	135	100	65	65	N	
C	110	LB	Concave, Rect.	R	120	115	104·3	138	123	146	110	67	67	A	
C	115	MB	Und., Elev.	R	105	112	93·7	132	113	114	92	56	55	A	
N	105	HB	Rect., Elev.	Nil	119	120	106·2	131	108	130	114	56	56	N	
C	105	MB	Rect., Hor.	R	115	116	99·1	130	108	133	104	65	67	N	
C	105	HB	Rect., Hor.	R	112	116	96·5	128	114	131	82	62	65	N	Hairs on tips of ears
N	120	LR	Concave, Elev.	R	115	111	103·6	130	108	127	102	55	61	A	
C	110	LR	Rect., Hor.	L	114	112	101·7	122	122	122	89	58	58	A	
C	115	MB	Und., Elev.	R	112	115	97·3	140	113	129	103	49	44	A	
C	110	MB	Und., Desc.	Nil	114	116	98·2	135	100	124	95	71	67	N	
C	105	MB	Concave, Elev.	Nil	121	121	100·0	127	92	123	112	56	60	N	
C	110	MB	Convex, Hor.	R	117	120	97·4	135	105	132	88	58	56	N	
C	105	MB	Concave, Hor.	R	116	119	97·4	125	98	134	97	59	59	A	
C	110	HB	Rect., Hor.	R	110	118	93·2	133	110	128	87	60	65	A	
N	110	MB	Rect., Desc.	L	118	115	102·6	124	116	136	85	58	62	A	
C*	110	H Projecting	Rect., Hor.	L	115	116	99·1	116	113	134	103	59	59	A	[of External Ears
C	110	LB	Rect., Elev.	R	112	120	93·2	125	113	125	97	50	48		Meati imperforate : Mere vestiges
C	112	MB	Rect., Elev.	R	116	113	102·6	133	107	134	88	59	65	A	
C	105	HB	Concave, Elev.	Nil	112	114	98·2	133	115	143	104	62	62	N	Darwin's Tubercle L.
C	110	MB	Rect., Elev.	L	122	120	101·6	121	111	137	104	63	62	N	
C	105	HB	Rect., Hor.	R	122	120	101·6	124	88	142	125	68	72	A	
C	105	HN	Convex, Elev.	R	121	113	107·0	120	95	130	106	63	66	A	Depression in sup. occip. region
C	110	MB	Convex, Hor.	R	122	125	97·6	125	92	145	127	66	63	A	
N	104	HB	Rect., Hor.	R	113	113	100·0	121	82	140	111	56	56	A	
N	102	HB	Convex, Hor.	R	121	115	105·2	120	104	133	108	62	64	N	
N	110	HB	Rect., Hor.	R	123	122	100·8	118	101	145	93	66	68	A	
C	110	B	Hump, Hor.	R	121	120	100·8	120	104	130	96	65	66	A	
C	110	LB	Hump, Elev.	R	128	125	102·4	140	122	148	98	61	60	A	
C	110	HB	Convex, Elev.	Nil	115	116	99·1	137	125	140	97	60	64	N	Malars prominent
C	115	LB	Rect., Elev.	R	115	117	98·3	127	115	135	93	60	57	N	
C	115	M	Concave, Elev.	R	110	105	104·7	137	114	122	95	58	60	N	
C*	108	MB	Hump, Elev.	R	117	116	100·8	115	102	132	103	58	59	A	
C	117	HB	Rect., Hor.	L	107	109	96·3	119	115	130	94	53	53	A	
C	110	MB	Convex, Elev.	R	110	115	95·6	116	103	135	90	57	61	N	

urrence slight.

Reg. and Crime	Cranium															Face					F
	Antero-posterior Curve	Ant. Segment	Post. Seg.	Horizontal Circum.	Right Ant.	R. Post.	L. Post.	L. Ant.	Ant. Seg.	Post. Seg.	Cranial Height	Cranial Length	Cranial Breadth	Cephalic Index	Vertical Index	Length	Breadth	Facial Index	Symmetry	Eye-brows. C = concurrent N = not concurrent	Curve
W 503 Murder ...	310	145	165	565	130	140	150	145	275	290	135	194	153	79·8	69·5	120	128	106·6	R+	C*	114
X 228 ,, ...	335	170	165	563	150	130	140	143	293	270	141	193	152	78·7	73·0	126	139	110·3	=	C	110
X 484 ,, ...	335	170	165	565	135	155	140	135	270	295	132	206	150	72·8	64·0	129	134	103·8	=	C	100
Y 43 ,, ...	334	180	154	570	130	150	150	140	270	300	136	202	145	71·7	67·3	120	129	107·5	R+	C	110
TX 930 ,, ...	320	164	156	560	140	145	135	140	280	280	132	196	151	77·0	67·8	102	135	132·3	R+	C	105
Y 95 ,, ...	315	170	145	560	145	155	130	130	275	285	141	198	150	75·6	71·2	134	128	95·5	=	C	110
Y 377 ,, ...	305	165	140	565	140	130	150	145	285	280	136	192	150	78·1	70·8	128	135	105·4	=	N	110
Y 514 ,, ...	335	175	160	565	135	145	153	127	262	298	138	198	150	95·7	69·6	126	136	107·9	R+	C	115
Z 46 ,, ...	315	160	155	530	135	145	130	120	255	275	138	190	145	76·3	72·6	117	136	116·2	L+	C	110
Z 270 ,, ...	340	180	160	550	150	135	135	130	280	270	138	195	149	76·3	70·7	126	135	107·1	=	N	112
a 432 ,, ...	320	160	160	580	135	150	165	130	265	315	133	195	160	82·0	68·2	126	146	115·8	=	C	108
Z 396 ,, ...	284	170	114	540	140	140	110	150	290	250	120	190	150	78·9	63·1	125	140	112·0	L+	C	117
Z 303 ,, ...	315	160	155	562	135	145	155	127	262	300	126	194	150	77·3	64·9	126	135	107·0	=	C	110
EZ 861 ,, ...	345	180	165	570	150	130	160	130	280	290	140	200	150	75·0	70·0	127	132	103·8	L+	C	110
N 260 Manslaughter...	310	157	153	560	140	145	140	135	275	285	135	190	154	81·0	71·0	122	136	110·5	L+	C	110
N 531 ,, ...	320	165	155	570	130	145	155	140	270	300	125	205	151	73·7	60·9	116	140	120·6	L+	C	110
R 219 ,, ...	310	170	140	570	140	145	145	140	280	290	128	202	154	76·2	63·3	130	144	110·7	L+	C	112
V 455 ,, ...	310	150	160	530	120	170	110	130	250	280	120	185	140	76·2	64·8	125	135	108·4	L+	C	110
b 466 Rape	300	165	135	555	140	140	135	140	280	275	135	194	150	76·9	69·5	130	140	107·6	=	C	110
MV 278 Rape ...	320	150	170	540	130	150	135	125	255	285	132	193	141	73·0	68·3	125	130	104·0	L+	C	110
U 559 ,, ...	310	160	150	560	140	130	155	135	275	285	127	185	157	84·8	68·6	120	138	115·0	L+	C	110
TV 580 Theft ...	320	170	150	550	145	125	145	135	280	270	133	186	152	81·7	71·5	128	140	109·5	=	C	110
b 319 ,, ...	315	150	165	560	130	150	150	130	260	300	120	190	153	80·5	63·1	127	143	112·5	L+	C	110
b 338 Burglary ...	340	175	165	570	140	150	160	120	260	310	150	195	154	78·9	76·9	120	133	110·8	L+	C	105
Y 366 Theft	325	175	150	540	130	160	115	135	265	275	130	182	145	79·6	71·4	122	133	109·0	L+	C	110
V 334 Forgery ...	320	170	150	555	140	150	130	135	275	280	132	190	148	77·8	69·4	135	136	107·0	L+	C	105
b 331 Theft	320	160	140	560	140	140	150	130	270	290	143	194	144	74·2	73·6	132	135	102·2	L+	N	110
b 262 Fraud	335	160	175	575	140	150	125	160	300	275	138	195	154	78·9	70·7	134	140	104·4	L+	C	110

* Concurrence slight.

O 595 Manslaughter..	340	170	170	580	150	145	155	130	280	300	138	206	156	75·7	66·9	147	144	97·9	L+	C	10
W 298 ,, ...	340	160	180	585	140	155	160	130	270	315	130	203	152	74·8	64·0	123	145	117·8	L+	C	11
L 371 ,, ...	335	160	175	545	140	135	150	120	260	285	138	194	142	73·1	71·1	130	140	107·6	R+	N	10
ZO 321 Wounding ...	300	155	145	530	140	130	130	130	270	260	120	185	155	83·7	64·8	125	150	120·0	L+	N	11
b 359 ,, ...	325	180	145	545	140	155	105	145	285	260	129	190	144	75·7	67·8	124	137	110·3	L+	C	11
L 280 ,, ...	315	140	175	540	140	145	135	120	260	280	114	185	150	81·0	61·6	124	138	111·2	L+	C	10
L 623 ,, ...	325	160	165	565	135	155	145	130	265	300	130	192	154	80·2	67·6	130	135	103·8	=	C	10
Z 257 ,, ...	330	170	160	550	140	140	145	125	265	285	130	192	146	76·0	67·6	125	138	110·4	L+	C	10
W 263 ,, ...	320	160	160	560	140	140	145	135	275	285	123	184	151	82·0	66·8	121	138	114·0	R+	C	10
P 320 ,, ...	340	170	170	595	150	155	165	125	275	320	148	210	156	74·2	70·4	125	134	107·2	L+	C	10
b 256 Rape	305	175	130	560	145	145	140	130	275	285	149	197	150	76·1	75·6	137	134	97·8	L+	C	10
Y 236 ,, ...	315	160	155	540	135	145	140	120	255	285	120	192	140	72·8	62·5	129	130	100·7	L+	C	11
W 423 ,, ...	300	165	135	530	135	140	130	125	260	270	127	180	144	80·0	70·5	116	135	116·3	L+	C	11
a 155 Bestiality ...	350	180	170	575	145	145	155	130	275	300	149	204	154	75·4	73·0	108	139	128·7	=	C	11
MQL 64 Coining ...	320	180	140	550	140	125	145	140	280	270	137	195	143	73·3	70·2	125	138	110·4	L+	C	10
a 63 Arson	335	160	175	565	150	150	140	125	275	290	134	193	147	76·1	69·4	119	139	116·8	=	N	11
P Theft	355	180	175	580	140	150	150	140	280	300	141	206	153	74·2	68·4	130	140	107·6	L+	C	10
DKTW 300 Theft ...	300	150	150	545	140	140	140	125	265	280	121	185	150	81·0	65·4	114	138	121·0	L+	C	11
b 106 Theft	310	155	155	555	145	140	135	135	280	275	118	187	153	81·8	63·1	135	140	103·6	L+	C	11
LFP 923 Theft ...	330	160	170	570	140	155	135	140	280	290	144	198	151	76·2	72·7	122	132	108·1	=	C	10
RL 454 ,, ...	335	170	165	535	130	140	140	125	255	280	127	185	146	78·9	68·6	128	133	103·8	L+	C	10
X 259 Theft	340	185	155	580	150	150	140	140	290	290	142	194	169	87·1	73·1	112	153	136·6	L+	C	10
LRWZ 268 Theft ...	325	160	165	550	180	95	145	130	310	240	126	186	155	83·3	67·7	115	138	120·0	L+	N	10
a 145 Theft	330	180	150	540	140	145	125	130	270	270	135	193	142	73·5	69·9	108	131	121·2	L+	C	10
TW 168 Burglary ...	340	170	170	560	130	150	145	135	265	295	129	194	155	79·8	66·4	123	131	106·5	L+	C	10
WL 419 ,, ...	320	150	170	565	140	155	135	135	275	290	133	200	148	74·0	66·5	125	141	112·8	=	C	11
BKX 463 ,, ...	325	155	170	555	140	140	150	125	265	290	126	190	153	85·0	66·3	122	141	115·5	L+	N	11
Z 367 ,, ...	320	160	160	555	145	130	145	135	280	275	116	190	149	78·4	61·0	112	133	118·7	R+	C	10
b 25 ,, ...	330	160	170	546	140	135	150	121	261	285	140	194	145	74·7	72·1	115	132	114·7	L+	C	10
Y 467 Fraudulent Enlistment	350	180	170	590	150	140	145	155	305	285	140	200	160	80·0	70·0	128	142	110·9	L+	C	10

TABLE I.—*continued.*

Eye-brows. C = concurrent N = not concurrent	Forehead: Curve	Forehead: High (H), Low (L), Receding (R), Med. (M), Broad (B)	Nose: General Shape	Nose: Deflection	Auriculo-Alveol. Diameter	Auriculo-Nasal Diameter	Gnathic Index	Auriculo-Occip. Diameter	Occipital Projection	Auriculo-Mental Diameter	Chin Projection	Ears: Length—Right	Ears: Length—Left	Ears: Lobule. A = adherent N = not adherent	General Observations
C*	114	LB	Und., Elev.	R	120	117	102·5	133	92	132	105	60	63	N	Deficient of last phalanx of fingers of R. hand, R. thumb normal. Index, mid and ring fingers of L. hand are rudimentary and fastened together as in a bag. Little finger and thumb normal
C	110	LB	Concave, Elev.	R	113	122	92·6	127	117	127	86	59	60	N	
C	100	LR	Rect., Elev.	R	127	122	104·0	132	110	146	122	65	64	N	
C	110	MB	Rect., Elev.	R	122	118	103·3	135	112	142	122	59	60	A	
C	105	HB	Rect., Hor.	R	113	120	94·1	117	105	133	109	64	66	A	Edentulous
C	110	MR	Hump, Elev.	R	120	113	106·1	137	113	129	105	57	60	N	[tulous
N	110	HB	Rect., Elev.	R	110	114	87·5	125	99	128	107	50	56	N	Tops of auricles turned over. Eden-
C	115	M Low	Concave, Elev.	R	113	118	95·7	134	115	133	87	58	60	A	
C	110	HN	Concave, Elev.	L	110	110	100·0	125	110	123	95	53	55	A	
N	112	ML	Concave, Elev.	Nil	122	108	112·9	127	120	140	87	58	62	A	
C	108	HB	Convex, Elev.	R	116	116	100·0	140	108	137	100	70	65	N	
C	117	HB	Rect., Elev.	R	122	122	100·0	137	102	143	119	59	62	N	
C	110	MB	Convex, Elev.	R	112	115	97·3	135	117	127	95	68	70	N	
C	110	LR	Concave, Elev.	R	125	120	104·1	125	97	135	100	57	60	A	Prom. Malars
C	110	HB	Rect., Hor.	L	108	107	99·0	135	100	134	95	53	62	A	Rudiment of Darwin's Tubercle
C	110	MB	Convex, Hor.	R	108	109	99·0	135	102	134	110	58	60	A	
C	112	LR	Concave, Elev.	R	119	117	101·7	136	115	133	95	66	70	N	
C	110	MB	Hump, Hor.	R	111	111	100·0	124	111	122	103	64	58	A	
C	110	HB	Convex, Hor.	Nil	116	122	95·0	130	104	125	118	62	62	N	
C	110	MB	Rect., Hor.	Nil	117	114	102·6	125	100	133	105	56	59	A	
C	110	LB	Rect., Hor.	R	118	118	100·0	118	97	135	96	53	55	A	
C	110	MB	Rect., Elev.	Nil	110	110	100·0	125	120	125	92	60	62	A	
C	110	LB	Concave, Hor.	R	120	126	95·2	127	110	134	84	51	53	A	Outstanding Ears
C	105	LB	Concave, Elev.	R	116	119	87·4	126	110	140	85	66	66	A	
C	110	L	Concave, Elev.	R	112	103	108·7	120	95	135	95	60	62	A	
C	105	LR	Rect., Hor.	R	115	117	98·2	129	107	135	110	60	57	A	
N	110	MB	Und., Hor.	R	112	115	97·3	134	115	122	85	60	65	N	
C	110	MB	Convex, Desc.	R	110	115	95·6	145	119	130	94	70	72	N	Very large lobes

urrence slight.

TABLE II.

C	100	—	—	R	123	120	102·5	138	106	133	88	70	69	—	Palate A
C	110	—	—	R	113	120	94·1	131	108	130	88	69	68	N	,, Low and Narrow
N	103	—	—	R	110	112	98·2	123	110	113	85	65	65	N	,, A
N	110	—	—	R	120	120	100·0	120	100	135	103	65	60	—	,, A
C	110	—	—	R	123	116	106·0	123	92	138	95	62	60	—	,, Low and Shallow
C	103	—	—	R	115	115	100·0	123	100	126	82	59	56	—	,, ,, ,,
C	105	—	—	L	108	114	94·7	125	90	126	85	55	58	N	,, A
C	105	—	—	R	112	110	101·8	120	112	128	93	60	58	N	,, Low and Narrow
C	105	—	—	R	106	108	97·1	117	102	124	118	58	64	—	,, A
C	105	—	—	L	122	120	101·6	134	117	137	97	65	67	A	,, Narrow and Shallow
C	100	—	—	R	107	110	102·8	133	127	125	97	67	62	—	,, A
C	110	—	—	R	110	109	100·8	120	106	126	72	60	57	A	,, A
C	110	—	—	R	107	118	90·6	130	92	126	83	53	55	A	,, Narrow and Shallow
C	110	—	—	R	108	112	96·4	124	112	128	97	66	65	N	,, Low and Shallow
C	103	—	—	R	117	121	96·6	125	100	130	105	60	53	—	,, A
N	110	—	—	Nil	110	112	98·2	130	112	121	92	59	58	—	,, High and Narrow
C	100	—	—	R	124	119	104·2	134	115	148	103	58	54	—	,, A
C	110	—	—	R	122	116	105·1	124	87	131	110	60	56	A	,, A
C	115	—	—	R	116	115	100·8	127	115	132	95	54	59	—	,, A
C	100	—	—	R	117	120	97·5	127	97	136	102	62	66	A	,, Narrow and Shallow
C	105	—	—	R	107	110	97·2	120	112	129	85	59	61	—	,, Narrow and High
C	105	—	—	L	111	119	93·2	117	112	130	89	52	60	A	,, Low
N	108	—	—	Nil	110	110	100·0	122	98	127	91	59	64	A	,, Shallow and Narrow
C	100	—	—	R	110	111	99·0	120	99	120	97	57	63	A	,, ,, ,,
C	103	—	—	R	116	112	103·5	124	102	133	90	55	57	—	,, Med. Height, Flat
C	113	—	—	L	121	124	97·5	126	107	128	97	59	62	—	,, A
N	110	—	—	R	112	115	97·8	122	107	124	92	68	62	N	,, A
C	105	—	—	R	108	114	94·7	123	111	120	85	55	57	A	,, A
C	105	—	—	R	113	110	102·7	122	107	129	90	55	58	A	,, Shallow and Narrow
C	105	—	—	R	122	117	104·2	130	110	138	106	66	63	—	,, A

A FIRST STUDY OF THE WEIGHT, VARIABILITY, AND CORRELATION OF THE HUMAN VISCERA, WITH SPECIAL REFERENCE TO THE HEALTHY AND DISEASED HEART.

By M. GREENWOOD, Junior.

1. During the last few years, thanks to many improvements in our methods of analysis, several of the biometric constants of the human body have been investigated. The coefficients of correlation of almost all the separate bones have been ascertained with some accuracy, and the suspected relationship between intellectual and physical characters has been closely scrutinised. Under these circumstances it is strange that so little work has been done on the weights and correlations of the viscera. Even such a simple measurement as the weight of the heart does not appear to have been calculated from any adequate series of observations. English text-books of anatomy give this weight on the authority of Reid and Peacock or on that of Clendinning. These two sets of observations are based upon very few cases. Peacock and Reid's results are drawn from 181 males and 110 females, and Clendinning's from 90 and 71 respectively*. It will be evident, therefore, that no great importance can be attached to them, even if we leave on one side the fact that they afford no materials for the study of correlation.

Clearly, the only way to obtain data for the solution of problems concerning the absolute and relative weights of the viscera is to extract as large a series of observations as possible from the *post-mortem* department records of a large general hospital. The present memoir contains the preliminary analysis of such a series from the pathological data of the London Hospital.

It might be supposed that *post-mortem* records would contain a very large number of available cases, and that the weights of the various organs would be found recorded with considerable accuracy. As a matter of fact, however, simple

* Clendinning's results are quoted in the English text-books, and also in Grisolle's *Traité de Pathologie Interne* (9th Ed. p. 200, Vol. II.). See also, K. Pearson: "Variation in Man and Woman" (*Chances of Death and other Studies in Evolution*, Vol. I. p. 316); Peacock and Reid: *London and Edinburgh Monthly Journal of Medical Science*, 1843–6, 1854.

measurements of this kind appear to be rather despised by *post-mortem* clerks, and the records are, from the quantitative point of view, very disappointing. Weights are frequently omitted altogether; sometimes we read "Spleen *about* 3 or 4 ozs."; at others, the ingenious writer appears to have given free play to his imagination, and we read of a man of forty-six years of age having a heart weighing *one* ounce!*

It is much to be desired that, in future, more accurate methods of recording these simple observations should be adopted, so that large numbers of valuable facts may be rendered available for statistical inquiry. In my own research, after excluding the large majority of the examinations, there yet remained a considerable number of fairly trustworthy data suitable for tabulation, and this paper contains some of the results deduced therefrom, which it is hoped will not be without interest. I propose to divide my subject into four parts:

First. I shall discuss the average sizes, variabilities, and correlations of the heart, liver, spleen, and kidneys in the general population, diseased and normal, to be found within a London general hospital.

Secondly. I shall consider only cases in which the organs were found healthy on *post-mortem* examination. We shall thus to some extent be able to appreciate the influence of disease in modifying the biometric constants of the organs in question.

Thirdly. I propose to deal with the influence of age on the biometric constants for the viscera in man.

And, lastly. I shall consider the influence of certain special diseases from the same standpoint. All the data dealt with in this memoir are for males, the number of females in my collection being very much smaller. As far as I am aware, no investigation of this kind on the viscera has yet been undertaken, and mine does not profess in any respect to be more than a preliminary study. Its object is to draw attention to the need of better *post-mortem* room records, and to indicate the wide field of valuable research which they open up, not only to the biometrician but to the physician†.

2. *The General Hospital Population.*

In my first series of tables I have dealt, subject to certain limitations, with a random sample of a general hospital population. To avoid the extreme changes due to youthful growth or senile decay I have tabulated only cases between the

* *L. H. Path. Reports*, 1899, No. 661.

† "Il ne faut jamais négliger de peser les organes, surtout ceux qui sont atteints de lésions pathologiques; le poids fournit souvent en effet des renseignements précieux sur le degré et sur l'importance des lésions; on n'oubliera pas cependant qu'il existe sur ce point des variations individuelles considérables..... Le poids total du sujet, la taille, l'âge, le sexe sont tout autant de conditions qui font varier le poids des organes eux-mêmes." (Bard: *Précis d'Anat. Path.* p. 736.)

ages of 25 and 55. Such a "random sample" is, of course, also a selection in that it consists solely of those patients who died in hospital, and upon whom a *post-mortem* examination was held. Thus it is far from being a random sample of the "general population" of the country, many classes of which are never found in the wards of a general hospital at all.

Evidently the population of a general hospital will chiefly consist of, (i) persons acutely ill, (ii) those suffering from surgical injuries or diseases, (iii) sufferers from medical affections requiring special treatment. Chronic maladies of old age, such as bronchitis, indeed, any highly chronic disease, will be under-represented in comparison with the general death-rate. Similarly the number of cases of valvular heart disease and rarer disorders, such as *Diabetes Mellitus* or *Insular Sclerosis* and other nervous lesions, will be above the general average.

Now, as pneumonia and bronchitis, particularly the latter, form a considerable number of the so-called "terminal affections" responsible for a large majority of all adult deaths, a random thousand necropsies will not give us the information we require as to the quantitative relations of average viscera, *post-mortem*. The error resulting from too few cases of senile bronchitis will be lessened, if not minimised, by the fact that we have confined our attention to cases of less than 55 years of age. But even thus we have too many cases of valvular cardiac disease, and as this affection tends to produce hypertrophy of the heart, the average weight in the first three tables is probably a good deal higher than that of the ordinary population at death.

It is, of course, to be remembered that this "general hospital population" does not mean the "normal" or healthy one. The above remarks are merely intended to show that a thousand deaths in hospital will not be due to exactly the same causes as a thousand deaths taken at random outside, and that therefore when we proceed to select sub-groups, such as "Normal Hearts," "Hearts in Pneumonia," etc., the material we have to select from is not what it would have been had we been able to start with 1000 random deaths in the population at large. And so, if we find that the average weight or variability of an organ is diminished when we proceed to special classes, we must bear in mind that possibly the change might not have been so striking if we had had a more representative sample to start from.

General Hospital Population.

TABLE I. *Hearts with Livers.* Number 1382.

Mean Heart 13·53 ozs.	Standard Deviation 4·680 ozs.
Mean Liver 63·01 ozs.	Standard Deviation 13·314 ozs.

Correlation of Heart and Liver $\cdot 1931 \pm \cdot 0175$.

TABLE II. *Hearts with Spleens.* Number 1303.

Mean Heart 13·07 ozs.	Standard Deviation 4·067 ozs.
Mean Spleen 6·61 ozs.	Standard Deviation 3·345 ozs.

Correlation of Heart and Spleen $\cdot 1827 \pm \cdot 0181$.

TABLE III. *Hearts with Kidneys.* Number 1293.

Mean Heart 13·14 ozs. Standard Deviation 4·134 ozs.
Mean Kidneys 12·68 ozs. Standard Deviation 3·125 ozs.
Correlation of Heart with Kidneys ·2577 ± ·0175.

In this table the weight is that of the two kidneys taken together. To these results I add a table of the coefficients of variation*, so as to obtain some appreciation of the relative variability of the organs in question.

TABLE A.

Relative Variability in Weights.

Organ	Coefficient of Variation
Hearts with Livers ...	34·59
Hearts with Spleens ...	31·12 mean 32·39
Hearts with Kidneys ...	31·47
Livers	... ... 21·12
Spleens	... ... 50·58
Kidneys	... ... 24·63

The substantial difference between the weights of the heart in the cases when livers were measured with hearts and the cases when either spleens or kidneys were measured is due to the fact that the 1382 cases of the former only in part cover the 1292 to 1303 cases of the latter, the additional cases, amounting to three or four hundred, are due to entries in which only two or three weights were given. It seemed desirable to include all possible cases in order to utilise as much material as possible. But there has clearly been some special reason for measuring livers in the case of very large hearts which has not arisen in the case of spleens or kidneys. Thus with livers we have hearts up to 36 ozs., but with either spleens or kidneys only up to 28 ozs.

On the whole with respect to both mean and variability, we may consider the hearts with spleens or kidneys to give a more reasonable approach to the biometric constants of the general hospital population than arises in the case of hearts with livers, where there is evidence of much more selection.

We notice at once:

(*a*) That the spleen is relatively much more variable than the heart, and the heart than the liver or kidneys.

(*b*) That the heart mean is considerably higher than that usually given in anatomical text-books†.

* The coefficient of variation = 100 × Standard Deviation ÷ Mean.

† Peacock and Reid's result on 181 male hearts is a mean of 10·699 ozs. The coefficient of variation calculated from their figures by Pearson is 19·825. For the liver, the mean (from 84 cases) is 53·48 and the coefficient of variation 14·32 (Pearson: *op. cit.* Vol. I. p. 316). It might, however, be better to compare these numbers with the results given later for the "healthy organs." Pearson says that we may probably conclude from Peacock's own statements that he "has cut off a considerable tail of really healthy hearts weighing over 12 ozs." (*op. cit.* p. 317).

(*c*) That there are quite sensible correlations between the weight of the heart and that of the other organs.

3. *The "Normal" Heart.*

Let us now consider the "normal" heart. Evidently the ideal normal heart is hardly at present capable of measurement with respect to any character other than those related to its manner of performing its functions. Such an organ ought really to be measured during the life of its owner, and we cannot do this, as we are unacquainted with the exact relation subsisting between body weight and heart weight in the living subject. The following is, perhaps, the best approximation to the truth that we are in a position to make. Correlation tables have been constructed for pairs of organs found to be healthy *post-mortem*. In any case in which I had the least reason to suspect the existence of disease, the measurements have been excluded.

The following results were reached:

Healthy Organs.

TABLE IV. *Hearts with Livers.* Number 358.

Mean Heart 11·04 ozs. Standard Deviation 1·923 ozs.
Mean Liver 60·44 ozs. Standard Deviation 8·948 ozs.
Correlation of Heart and Liver ·2780 ± ·0329.

TABLE V. *Hearts with Spleens.* Number 517.

Mean Heart 11·25 ozs. Standard Deviation 2·073 ozs.
Mean Spleen 5·22 ozs. Standard Deviation 1·996 ozs.
Correlation of Heart and Spleen ·2654 ± ·0276.

TABLE VI. *Hearts with Kidneys.* Number 413.

Mean Heart 11·24 ozs. Standard Deviation 1·946 ozs.
Mean Kidney* 12·01 ozs. Standard Deviation 2·016 ozs.
Correlation of Heart and Kidneys ·4004 ± ·0279.

Drawing up a table of coefficients of variation as before we have:

TABLE B.

Relative Variability of Healthy Organs.

Organ	Coefficient of Variation
Hearts with Livers ...	17·42
Hearts with Spleens ...	18·42 } mean 17·71
Hearts with Kidneys ...	17·30
Livers	... ... 14·80
Spleens	... ... 38·21
Kidneys	... ... 16·80

* The mean value of the *right* kidney of 100 males 20 to 55 years of age, as deduced by Pearson from Reid and Peacock's values, is 5·57 ozs.

From these results we infer,

(*a*) That the average "healthy" organs are all lighter than those of the average general hospital population, and probably lighter than those of the general population as a whole. The weights are still, however, higher than those given by Peacock and Reid or by Clendinning.

(*b*) In passing from the mixed hospital population to the class of healthy organs, we find in every case the absolute variability is reduced, and by very large amounts indeed, the variability of the heart by over 50 p.c., and the other organs by amounts even 30 to 40 p.c. of their value.

(*c*) Relatively, the healthy spleen is still the most variable organ, and the heart comes second, but the kidneys are now close to the heart and the liver not very far behind. Disease appears to affect the weights of heart and spleen most, of livers and kidneys least.

We notice that our value for the coefficient of variation of the healthy heart is now 17·7 as against the result deduced from Peacock and Reid's measurement of 19·8 and for the liver 14·8 as against their 14·3*. We could hardly have anticipated such good agreement, and it certainly tends to confirm the value of the coefficient of variation as a fairly "steady" biometric constant.

4. *Influence of Age on the Weights of the Viscera.*

I have first considered the change in the absolute weight of the adult male heart with age, and I have then investigated the influence of age upon one set of correlations and variabilities, i.e. those of heart and spleen.

Table VII. gives the correlation between age and weight of heart in the case of health. We deduce the following values of the constants:

TABLE VII. *Relationship of Weight of Healthy Heart to Age.* Number 699.

Mean Heart 11·13 ozs.	Standard Deviation 2·015 ozs.
Mean Age 40·23 years.	Standard Deviation 8·500 years.

Correlation of Weight of Heart with Age = ·1363 ± ·0250.

There is thus a distinctly sensible increase of heart weight in health with age, the coefficient of correlation is more than five times its probable error. Still the correlation is smaller than what we might possibly have anticipated. Calculating the regression line we have, if H_p be the probable weight of heart in ozs., and A the age in years:

$$H_p = 9{\cdot}8322 + {\cdot}0323\,A.$$

Thus the average healthy heart gains about $\frac{1}{3}$ oz. per ten years. For example we have:

* Pearson (*The Chances of Death*, Vol. I. p. 318) gives 20·49 for the coefficient of variation of the *right* kidney as deduced from Peacock and Reid's measurements, as against my values of 16·80 for healthy and 24·63 for general hospital weights of both kidneys.

Average weight of Heart at 20 years is 10·48 ozs.
" " " 30 " " 10·80 "
" " " 40 " " 11·12 "
" " " 50 " " 11·45 "
" " " 60 " " 11·77 " *

As far as I am aware, the correlation between age and weight of body in adult males is unknown. We should *à priori* expect it to be greater than the above, but it is always dangerous to judge without actual data. The correlation between stature and age is known, and is actually *negative*; the stature of the adult male diminishing by about $\frac{1}{3}$ inch for ten years after his prime at 28 years†. We thus conclude that while the stature decreases by $\frac{1}{3}$ unit, the heart increases by $\frac{1}{3}$ unit per ten years.

When we remember that the healthy heart is on the average much smaller than the heart in disease, and that sickness on the average increases continuously with age‡, we shall probably lay less emphasis on the general *à priori* idea that the weight of the adult heart increases very sensibly with age alone.

For my second inquiry I had unfortunately not sufficient material to divide my healthy hearts into three age groups and thus determine the influence of age on variability and correlation. I should have got less than 180 cases for each table. I was thus reluctantly compelled to deal with hearts in the general hospital population.

I divided them into three groups, ages 25—35, 35—45, and 45—55. The following results were reached:

Age Influence on Hearts and Spleens of General Hospital Population.

TABLE VIII. *Hearts and Spleens.* Ages 25—35. Number 358.

Mean Heart 11·91 ozs. Standard Deviation 3·997 ozs.
Mean Spleen 7·45 ozs. Standard Deviation 3·758 ozs.
Correlation of Heart and Spleen = ·0785 ± ·0384.

TABLE IX. *Hearts and Spleens.* Ages 35—45. Number 536.

Mean Heart 13·16 ozs. Standard Deviation 4·018 ozs.
Mean Spleen 6·60 ozs. Standard Deviation 3·428 ozs.
Correlation of Heart and Spleen = ·1817 ± ·0282.

TABLE X. *Hearts and Spleens.* Ages 45—55. Number 403.

Mean Heart 13·65 ozs. Standard Deviation 4·425 ozs.
Mean Spleen 6·21 ozs. Standard Deviation 3·120 ozs.
Correlation of Heart and Spleen = ·2518 ± ·0315.

* Clendinning gives, Ages 15—29, 8½ ozs.; 30—50, 9½ ozs.; 50—60, 10¼ ozs. *Med. Chirurg. Trans.* 1838. See also Peacock and Reid, *op. cit.*

† *Biometrika*, Vol. I. pp. 46–9.

‡ *Biometrika*, Vol. II. pp. 260 *et seq.*

Forming as before a table of coefficients of variation, we have:

TABLE C.

Relative Variabilities of General Hospital Population of Hearts and Spleens at different Ages.

Organ	Coefficient of Variation	Organ	Coefficient of Variation
Heart, 25—35	32·79	Spleen, 25—35	50·42
Heart, 35—45	30·53	Spleen, 35—45	51·97
Heart, 45—55	32·42	Spleen, 45—55	50·24

We can draw some important results from the above constants.

(*a*) The heart in the general hospital population of adults increases far more rapidly with age than it does in the class of healthy hearts. On the other hand the weight of the spleen sensibly decreases.

(*b*) The absolute variability of the heart increases 10 per cent., and the absolute variability of the spleen decreases 17 per cent. during the period considered. These are quite sensible changes. Thus, while the heart tends to grow larger and more variable, the spleen tends to grow smaller and less variable.

(*c*) If we deal with relative variation as judged by the coefficient of variation, we see that the changes referred to under (*a*) and (*b*) almost balance each other. Or, the relative variabilities of both heart and spleen remain sensible constants with age and equal to the values found for the general hospital population of adults of all ages in Table A, p. 66. This is further evidence of the real value of the coefficient of variation as a biometric measure of variability.

(*d*) The correlation between heart and spleen steadily increases with age. In the first period it is comparatively small, in the second period it has much the same value as in the general hospital population of adults (see p. 65), and in the third period it approaches the value found for healthy adults.

These results are quite reasonable. As death below the age of 35 is generally abnormal*, we should expect to find that the coefficient of correlation was low. Over the age of 45 years there is a slow deterioration of all organs. There is no evidence to show that this degeneration is much more acute in any one of the organs we are considering than in any other. Therefore, although the absolute weights will differ from the normal, the correlation may be the same, and this we see that it actually is.

* By "abnormal" is meant here a death due to disease; the result of an accident would be from this point of view a normal death, as probably leaving the viscera "healthy" under *post-mortem* record.

5. *Influence of Special Diseases on the Cardiac Biometric Constants.*

We are now in a position to consider the effect of some special diseases on the weight and correlations of the heart. Unfortunately, scantiness of material and pressure on my time hindered my developing this most interesting branch of my subject very fully in the present paper. I shall hope to give it further consideration in another communication. I have confined my attention here to the heart and spleen weights, variabilities, and correlations in the case of two disease groups. First, Pneumonia (excluding tubercular disease); secondly, Valvular Disease of the Heart and Aortic Aneurism. I regret that the total number of cases available is small.

TABLE XI. *Hearts and Spleens. Cases of Pneumonia.* Number 177.

Mean Heart 12·50 ozs. Standard Deviation 2·768 ozs.
Mean Spleen 6·59 ozs. Standard Deviation 2·842 ozs.
Correlation of Heart and Spleen = ·1065 ± ·0501.

TABLE XII. *Hearts and Spleens. Cases of Valvular Disease and Aortic Aneurism.* Number 166.

Mean Heart 19·08 ozs. Standard Deviation 5·950 ozs.
Mean Spleen 8·57 ozs. Standard Deviation 5·158 ozs.
Correlation = ·0552 ± ·0522.

Forming as before a table for relative variabilities:

TABLE D.

Relative Variabilities of Heart and Spleen under Special Diseases.

Disease	Organ	Coefficient of Variation
Pneumonia	Heart	22·15
Valvular Disease and Aortic Aneurism	Heart	31·18
Pneumonia	Spleen	43·12
Valvular Disease and Aortic Aneurism	Spleen	60·16

From these values of the constants we may draw the following conclusions:

(*a*) In cases of pneumonia the mean weight of the heart is above that of the healthy heart (p. 67), but slightly below that of the general hospital population heart (p. 65). The weight of the spleen is somewhat above that of the healthy spleen (p. 67), and only about equal to that of the general hospital population spleen (p. 65). The absolute variabilities of both heart and spleen in pneumonia are far lower than the values in the general hospital population, and only slightly higher than their values in the case of healthy organs. The same remark applies to relative variabilities, which are in the case of this disease somewhat higher than the healthy values, but considerably below those of the general hospital population. We must therefore conclude that pneumonia does not influence in a *marked* degree the average values or the variabilities of either heart or spleen weights. As it

does raise the variability somewhat, i.e. introduces disturbances in the relationship, we are not surprised to find that it weakens, although again not in a very marked degree, the correlation between weights of heart and spleen.

(*b*) On the other hand, Valvular Disease and Aortic Aneurism send up the weights of *both* heart and spleen not only vastly above their healthy values, but markedly above the values for the general hospital population. Further, their absolute variabilities are increased considerably above the general hospital population values, and *à fortiori* above the healthy values. The coefficients of variation of both are also far above the values in health, and that for the spleen above the general hospital values. In the case of the heart, the mean has been sent up so high that although the absolute variability is considerably greater, the relative variability remains much the same. The general effect of these heart diseases is to render the correlation between heart and spleen hardly sensible.

If we may judge by these two cases the general effect of disease is to increase the variability of affected organs and reduce their correlation. This is absolutely in keeping with the sensible, but of course less marked, changes we find when we pass from a population with healthy organs to the general hospital population, which of course contains much disease.

6. *General Conclusions.*

The present paper is chiefly intended as an illustration of how effective biometric methods might be from the standpoint of medical science if only there were a systematic collection on a large scale of normal and pathological data. Some definite conclusions, however, may yet be drawn, and some suggestions made on the basis of our numbers. We see sensible, if moderate, correlations between the weights of heart, liver, spleen, and kidneys.

It may be somewhat difficult to understand why the heart-kidney correlation is higher than that of the heart with any of the other organs. That the kidney should be more closely associated with the heart than the liver is possibly owing to its more subordinate functions. The liver is the seat of so many important processes that its immediate connection with the heart is not so great as that of the heart with the kidney. The excretion of fluids is so closely bound up with physical changes in the vascular system, and conversely changes in renal structure react so markedly upon the heart and blood vessels that a very close physical relationship appears probable. No doubt the liver is greatly affected in many forms of cardiac disease, but on the other hand serious functional disturbances or even acute inflammation of the liver do not produce heart changes with the same precision as analogous mischief in the kidney does. If this partially and imperfectly accounts for the higher heart-kidney correlation as compared with the heart-liver coefficient, it may perhaps serve as an argument in the case of the spleen. Heart mischief nearly always reacts on the spleen, but splenic trouble does not always affect the heart.

If we consider in broad lines the general results of our investigation, we should say that they may be summed up in the statement that both special diseases and the general want of health to be found in a hospital population tend in the same directions, namely, to increase the variability of the organs dealt with and to reduce their correlation. As we pass from the general hospital population to a healthy population we find that variability sinks and correlation rises. To what extent this is an antecedent or concomitant of the diseased state it might not be always possible to assert.

In taking any population, low variabilities and high correlations are the two factors which measure closeness to type. As a general rule, under a given environment closeness to type is a condition of stability, we may almost say, of low selective death-rate. Hence we may look upon disease as less stringent approach to type, and high variability and low correlation as a sign of instability *.

Of course the capacity to vary absolutely and to alter the relationship of organs must exist, or a race will not be able to effect a change in type with a changing environment. Still, if we trust the theory of correlation by natural selection at all, death before senility as far as it is selective is the destruction of the less fit, i.e. of those not approaching within certain limits the type suitable to the environment. Thus it comes about that we shall expect on the Darwinian theory to find the individuals who die of disease in adult life to be more variable and less highly correlated in their organs than the "healthy." This is precisely what we do find, and the *post-mortem* room provides direct evidence in favour of the action of natural selection in the case of man. Indeed, in a not very conscious way the medical world has been expressing these very truths of evolution in other words. The figures we have considered showing lowered correlation in the diseased state are a biometric illustration of the truth of Dr Sutton's aphorism, "Disease is absence of rhythm†." In the normal or healthy group we see a population possessing the characteristic marks of stability, small variability, and high correlation. In the two special and the general diseased groups we have conditions tending in the opposite sense. In the healthy class we get a closer quantitative relationship between the weights of the viscera, in the diseased there is greater variety of proportions. Indeed, to adopt a well-worn definition, "Among the diseased each organ has a life and growth of its own, irrespective of the needs of the organism as a whole." Our biometric investigation shows us this independent life and growth leading to increased variability and to lessened correlation, shortly, to those deviations from type which beyond certain limits are incompatible with survival under a given environment ‡.

* In the evolution of subspecies it seems probable that the hardiest and most prolific groups have the least coefficients of variation. Thus in an investigation recently made by Mr A. Bacot and the author on the variability of *Spilosoma Urticae*, it was found that in several series of broods the groups with the largest coefficients of variation had the least net fertility.

† *Medical Pathology*, 1886, p. 95 *et seq.*

‡ I desire to take this opportunity of expressing my gratitude to Prof. Karl Pearson, to whose staff, among other acts of kindness, I owe the correction of many arithmetical slips in the above results. Anything of interest in this essay is due, either directly or indirectly, to him.

TABLE I.

General Hospital Population. Males. Hearts and Livers.

Hearts in ozs.

Livers in ozs.	5·125— 6·125	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	28·125—29·125	29·125—30·125	30·125—31·125	31·125—32·125	32·125—33·125	33·125—34·125	34·125—35·125	35·125—36·125	Totals
28·125— 32·125	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
32·125— 36·125	1	—	1	—	5	1	2	2	—	—	—	—	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16
36·125— 40·125	1	2	5	5	4	4	1	—	1	—	1	1	1	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	27
40·125— 44·125	4	1	2	8	5	4	3	5	2	2	1	—	—	1	—	1	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	41
44·125— 48·125	—	3	5	9	11	13	3	4	4	1	4	2	5	1	1	—	3	—	1	2	—	1	—	—	—	—	—	—	—	—	—	73
48·125— 52·125	2	1	4	13	18	15	19	6	9	4	5	—	2	3	5	3	1	—	2	1	—	—	—	—	—	—	—	—	—	—	—	113
52·125— 56·125	—	4	7	7	19	15	26	28	8	10	12	4	3	4	2	1	4	3	6	—	2	3	—	—	—	—	—	—	—	1	1	170
56·125— 60·125	—	3	9	12	19	23	17	17	14	17	14	2	13	5	2	2	1	3	1	1	1	1	—	—	1	—	—	—	—	—	—	178
60·125— 64·125	—	—	5	8	24	30	22	17	18	15	12	4	5	5	8	—	2	3	3	1	2	—	—	—	—	1	—	—	—	—	—	185
64·125— 68·125	—	4	3	5	10	21	22	24	14	2	8	4	5	3	5	—	3	1	1	1	—	1	1	1	—	—	—	—	—	—	—	139
68·125— 72·125	—	—	4	3	12	12	19	8	12	7	15	4	4	5	2	1	2	6	1	2	—	—	2	—	—	—	1	—	1	—	—	123
72·125— 76·125	—	2	—	2	9	5	13	9	16	7	12	4	2	3	2	1	1	—	1	1	1	—	2	—	—	—	—	1	2	—	—	96
76·125— 80·125	—	—	1	2	7	12	10	7	10	3	7	3	3	1	3	1	1	—	2	—	1	1	—	—	—	—	—	—	—	—	—	75
80·125— 84·125	—	1	—	1	5	1	7	—	7	2	3	3	—	3	—	1	1	—	—	—	1	1	—	1	—	—	—	—	—	—	—	38
84·125— 88·125	—	—	—	—	1	4	4	6	5	6	1	1	2	1	1	2	—	1	3	—	—	—	—	2	—	—	—	1	—	—	—	41
88·125— 92·125	—	—	—	1	—	2	4	—	4	5	1	3	1	—	—	—	—	—	2	—	—	2	—	—	—	—	1	—	—	—	—	26
92·125— 96·125	—	—	2	—	2	—	3	4	—	1	1	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	1	—	—	—	—	17
96·125—100·125	—	—	—	—	—	—	6	5	2	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16
100·125—104·125	—	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	7
Totals ...	8	22	48	76	151	162	182	143	127	83	98	37	50	36	33	13	21	17	24	9	8	11	6	4	1	1	4	2	3	1	1	1382

TABLE II.

General Hospital Population. Males. Hearts and Spleens.

Hearts in ozs.

Spleens in ozs.	5·125— 6·125	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	Totals
1·125— 2·125	2	2	3	4	3	4	4	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	27
2·125— 3·125	1	4	6	9	16	15	9	8	3	1	3	—	2	—	2	—	1	—	1	—	—	1	—	82
3·125— 4·125	5	5	7	20	26	21	19	19	13	14	13	3	7	1	2	2	2	1	—	—	—	2	—	180
4·125— 5·125	—	4	5	17	34	26	24	18	23	12	17	4	6	2	3	5	1	1	3	1	—	2	—	208
5·125— 6·125	—	3	10	12	24	37	36	23	22	6	16	6	7	7	7	1	2	3	6	—	—	1	2	231
6·125— 7·125	—	—	1	5	21	16	16	12	7	8	7	2	3	5	6	—	2	1	3	2	1	1	—	119
7·125— 8·125	—	—	4	4	11	14	23	18	12	9	13	8	6	3	5	—	3	2	3	1	—	2	—	141
8·125— 9·125	—	1	2	3	5	13	11	10	9	9	3	2	4	1	2	—	1	1	—	1	1	—	1	80
9·125—10·125	—	—	1	1	7	5	12	8	9	5	5	1	5	1	4	1	2	—	1	2	1	1	—	72
10·125—11·125	—	—	1	4	4	2	2	—	5	1	—	2	2	—	1	—	2	—	1	1	—	1	—	29
11·125—12·125	—	1	—	2	6	1	7	4	3	5	2	1	1	2	—	1	—	1	2	—	2	—	—	41
12·125—13·125	—	—	—	—	—	3	1	2	1	3	1	1	—	—	—	—	—	1	—	—	—	—	2	15
13·125—14·125	—	—	1	—	2	3	6	1	2	—	4	—	1	1	—	—	—	—	1	—	—	—	—	22
14·125—15·125	—	—	2	1	1	1	—	—	—	1	2	2	2	—	1	—	—	—	—	—	1	—	—	14
15·125—16·125	—	1	1	—	1	1	5	2	1	—	1	1	1	3	1	—	1	—	—	—	—	—	—	20
16·125—17·125	—	—	1	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
17·125—18·125	—	—	—	—	1	1	2	—	—	1	—	—	1	—	1	1	—	1	1	—	—	—	—	10
18·125—19·125	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
19·125—20·125	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
20·125—21·125	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
21·125—22·125	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
22·125—23·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals ...	8	19	45	83	162	166	181	127	114	76	88	33	48	26	35	11	17	12	22	8	6	11	5	1303

TABLE III.

General Hospital Population. Males. Hearts and Kidneys.

Hearts in ozs.

Kidneys in ozs.	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	Totals
6·125— 7·125	2	2	2	3	4	1	2	1	—	1	—	1	2	1	—	—	—	—	1	1	—	—	24
7·125— 8·125	4	—	7	7	5	6	5	2	—	4	—	3	3	1	—	—	—	1	—	—	—	—	48
8·125— 9·125	3	6	7	11	6	5	5	2	4	2	1	2	1	3	2	1	1	1	—	—	2	1	66
9·125—10·125	2	11	11	13	29	14	9	9	4	9	3	2	1	1	1	4	—	2	—	1	—	—	126
10·125—11·125	4	2	15	34	25	20	9	5	4	3	2	2	2	1	2	1	2	—	—	1	1	—	135
11·125—12·125	4	7	16	24	38	34	26	16	15	8	4	8	5	2	1	2	—	1	1	1	—	1	214
12·125—13·125	2	2	9	23	20	20	19	20	4	12	4	6	3	3	1	1	1	2	1	—	—	—	153
13·125—14·125	—	4	8	13	16	28	20	14	12	10	7	5	4	7	1	1	5	7	2	—	4	—	168
14·125—15·125	—	1	5	12	5	16	11	17	6	10	2	3	1	6	1	2	3	1	2	—	—	—	104
15·125—16·125	—	3	2	10	10	13	11	12	7	13	2	7	1	—	2	2	—	2	—	1	1	3	102
16·125—17·125	—	—	1	3	4	11	6	2	4	8	4	2	—	—	—	1	—	1	—	—	—	—	47
17·125—18·125	—	1	1	3	3	3	7	8	2	3	—	3	3	2	—	—	1	3	—	—	—	—	43
18·125—19·125	—	—	—	2	3	1	1	2	2	1	3	—	—	2	—	1	1	1	—	1	—	—	21
19·125—20·125	—	—	—	2	1	3	3	1	1	—	1	1	—	1	—	—	1	1	1	—	—	—	17
20·125—21·125	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	3
21·125—22·125	—	—	—	—	1	—	—	—	2	1	1	—	—	1	—	—	—	1	—	—	—	—	7
22·125—23·125	—	—	—	—	—	—	—	1	1	—	2	1	—	1	—	—	1	—	—	—	—	—	7
23·125—24·125	—	—	—	—	—	1	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	4
24·125—25·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
25·125—26·125	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2
26·125—27·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
27·125—28·125	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals ...	21	39	84	160	170	177	135	112	69	88	37	46	27	32	11	17	16	24	9	6	8	5	1293

TABLE IV.

Healthy Hearts and Livers. Males.

Hearts in ozs.

Livers in ozs.	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	Totals
44·125—48·125	3	5	3	5	3	2	1	—	—	22
48·125—52·125	2	12	13	10	7	3	3	2	2	54
52·125—56·125	3	8	15	6	10	9	1	1	—	53
56·125—60·125	6	8	11	13	7	6	4	3	3	61
60·125—64·125	2	3	12	9	10	2	7	2	1	48
64·125—68·125	1	3	3	13	13	6	5	1	—	45
68·125—72·125	1	1	4	4	7	4	2	1	2	26
72·125—76·125	—	1	1	4	5	6	7	1	2	27
76·125—80·125	—	1	3	6	3	3	5	1	—	22
Totals ...	18	42	65	70	65	41	35	12	10	358

TABLE V.

Healthy Hearts and Spleens. Males.

Hearts in ozs.

Spleens in ozs.	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	Totals
1·125— 2·125	4	3	3	2	2	1	2	1	—	—	18
2·125— 3·125	6	7	12	9	6	5	1	—	—	—	46
3·125— 4·125	6	15	21	16	14	10	6	11	2	—	101
4·125— 5·125	1	12	25	18	15	11	13	3	5	1	104
5·125— 6·125	5	8	16	27	24	13	13	—	5	—	111
6·125— 7·125	1	5	9	4	10	9	2	2	2	—	44
7·125— 8·125	—	2	7	8	6	11	8	4	6	1	53
8·125— 9·125	—	1	2	7	1	3	1	1	—	1	17
9·125—10·125	—	—	—	1	5	—	2	2	3	—	13
10·125—11·125	—	2	1	1	—	—	—	—	—	—	4
11·125—12·125	—	—	—	—	2	1	1	1	—	1	6
Totals ...	23	55	96	93	85	64	49	25	23	4	517

TABLE VI.

Healthy Hearts and Kidneys. Males.

Hearts in ozs.

Kidneys in ozs.	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	Totals
7·125— 8·125	1	3	—	2	2	1	—	—	—	9
8·125— 9·125	1	6	5	1	3	1	—	—	—	17
9·125—10·125	7	7	6	14	6	4	3	1	1	49
10·125—11·125	2	8	23	17	11	4	2	2	1	70
11·125—12·125	1	5	11	22	19	12	7	3	1	81
12·125—13·125	1	4	17	12	11	13	8	1	1	68
13·125—14·125	3	2	8	7	12	9	7	4	2	54
14·125—15·125	1	—	4	3	5	5	8	2	5	33
15·125—16·125	1	1	1	1	4	2	7	4	3	24
16·125—17·125	—	—	—	1	2	1	—	2	1	7
17·125—18·125	—	—	—	—	1	—	—	—	—	1
Totals ...	18	36	75	80	76	52	42	19	15	413

TABLE VII.

Correlation of Age and Heart Weight. Healthy Males.

Hearts in ozs.

Ages. Years	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	Totals
25th and 26th	3	4	7	6	3	2	5	1	—	—	31
27th and 28th	3	9	10	9	8	3	1	2	—	—	45
29th and 30th	3	6	10	9	12	7	2	1	1	—	51
31st and 32nd	4	7	9	9	6	4	6	—	—	—	45
33rd and 34th	5	4	9	14	7	7	2	3	3	—	54
35th and 36th	2	6	4	11	10	8	1	1	3	—	46
37th and 38th	3	7	10	6	10	5	2	1	4	1	49
39th and 40th	—	5	13	13	16	10	4	1	3	—	65
41st and 42nd	—	3	11	4	7	8	4	3	3	—	43
43rd and 44th	—	3	8	4	8	3	6	—	2	—	34
45th and 46th	4	4	13	7	10	5	8	3	3	—	57
47th and 48th	1	3	7	4	5	7	4	1	—	—	32
49th and 50th	1	7	13	5	5	7	5	6	—	1	50
51st and 52nd	3	4	7	7	6	2	6	4	3	—	42
53rd and 54th	—	4	8	13	7	7	4	—	3	1	47
55th and 56th	1	1	1	—	2	1	1	1	—	—	8
Totals ...	33	77	140	121	122	86	61	28	28	3	699

TABLE VIII.

Hearts and Spleens. General Hospital Population. Males. Young Adults (25—35 years).

Hearts in ozs.

Spleens in ozs.	5·125— 6·125	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	Totals
1·125— 2·125	—	1	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
2·125— 3·125	1	1	2	1	6	3	—	1	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	16
3·125— 4·125	2	1	4	5	6	6	6	3	2	1	2	—	—	1	—	2	—	—	—	—	—	—	—	39
4·125— 5·125	—	1	1	7	5	7	6	5	4	5	2	—	—	1	—	—	2	—	2	—	—	—	1	46
5·125— 6·125	—	—	5	3	6	9	9	5	8	1	3	1	3	1	—	—	—	—	—	1	1	1	—	59
6·125— 7·125	—	—	—	2	10	5	7	2	1	2	1	1	—	1	—	—	—	—	1	1	—	—	—	35
7·125— 8·125	—	—	1	5	5	7	11	5	—	2	2	1	1	1	1	—	—	—	1	1	1	—	—	44
8·125— 9·125	—	—	1	2	2	5	1	4	1	—	1	—	1	—	—	—	—	—	—	1	—	—	—	20
9·125—10·125	—	1	—	1	6	4	4	3	1	1	2	—	—	—	1	—	1	—	—	1	—	1	—	26
10·125—11·125	—	—	—	1	3	—	—	—	3	1	—	1	1	—	—	—	—	—	1	—	—	—	—	12
11·125—12·125	—	1	—	1	3	1	3	3	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	17
12·125—13·125	—	—	—	—	—	2	1	—	—	1	—	—	—	—	—	—	—	1	1	—	—	—	1	7
13·125—14·125	—	—	1	—	—	2	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	6
14·125—15·125	—	—	2	1	2	—	—	—	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	9
15·125—16·125	—	—	—	—	1	1	2	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	5
16·125—17·125	—	—	1	—	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
17·125—18·125	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
18·125—19·125	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
19·125—20·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20·125—21·125	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
21·125—22·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
22·125—23·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
23·125—24·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals ...	3	6	18	32	56	56	57	31	25	16	16	6	7	5	2	3	3	1	5	4	2	2	2	358

TABLE IX.

Hearts and Spleens. General Hospital Population. Males. Prime of Life (35—45 *years*).

Hearts in ozs.

Spleens in ozs.	5·125— 6·125	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	28·125—29·125	Totals
1·125— 2·125	1	2	1	1	3	2	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
2·125— 3·125	—	—	3	5	5	6	6	5	2	—	2	—	1	—	1	—	—	—	—	—	—	—	—	—	36
3·125— 4·125	1	2	1	6	8	7	7	12	6	5	8	1	4	—	2	1	2	1	—	—	—	—	—	—	74
4·125— 5·125	—	2	3	6	16	10	11	10	9	4	8	1	3	—	2	2	2	2	2	1	—	2	—	—	96
5·125— 6·125	—	2	2	6	10	18	16	6	6	4	4	2	1	3	3	1	1	—	—	—	—	—	—	—	85
6·125— 7·125	—	—	—	1	7	9	3	7	5	3	3	1	3	2	2	—	3	1	—	—	—	—	—	—	50
7·125— 8·125	—	—	—	1	4	5	8	3	8	3	3	5	2	2	2	—	1	2	2	—	—	—	—	—	51
8·125— 9·125	—	1	1	—	—	4	9	5	4	5	1	2	2	—	1	—	—	—	—	—	—	—	—	—	35
9·125—10·125	—	—	1	2	2	—	4	4	5	3	4	—	1	—	1	—	1	—	1	—	1	—	—	—	30
10·125—11·125	—	—	1	2	1	1	2	—	1	1	1	—	—	—	1	—	—	—	—	1	—	—	—	—	12
11·125—12·125	—	—	—	1	2	—	2	1	—	—	2	1	1	—	—	—	—	1	—	—	1	—	—	1	13
12·125—13·125	—	—	—	—	—	1	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	5
13·125—14·125	—	—	—	—	1	1	4	1	2	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	12
14·125—15·125	—	—	—	—	—	1	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	3
15·125—16·125	—	1	1	—	—	—	1	2	—	—	1	1	—	1	1	—	1	—	—	—	—	—	—	—	10
16·125—17·125	—	—	1	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
17·125—18·125	—	—	—	—	—	1	—	—	—	1	—	—	1	—	1	—	—	1	1	—	—	—	—	—	6
18·125—19·125	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
19·125—20·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20·125—21·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
21·125—22·125	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals ...	2	10	15	31	59	67	76	58	50	31	39	15	20	8	18	4	11	8	6	2	2	2	1	1	536

TABLE X.

Hearts and Spleens. General Hospital Population. Males. Middle Age (45—55 years).

Hearts in ozs.

Spleens in ozs.	5·125— 6·125	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	28·125—29·125	Totals
1·125— 2·125	1	1	3	2	2	1	3	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
2·125— 3·125	—	1	1	3	6	5	3	1	—	—	1	—	1	—	—	—	2	—	1	—	—	1	—	—	26
3·125— 4·125	—	—	1	9	10	7	4	7	6	7	3	1	3	1	—	—	—	—	—	—	—	1	—	—	60
4·125— 5·125	—	1	1	4	15	9	5	4	10	4	8	4	1	1	1	—	—	—	1	—	—	1	—	—	70
5·125— 6·125	—	2	2	—	6	10	12	10	6	2	7	2	3	3	3	—	—	1	3	—	—	1	1	—	74
6·125— 7·125	—	—	1	2	3	2	6	3	2	1	4	1	—	2	4	—	—	—	4	1	—	—	—	—	36
7·125— 8·125	—	—	2	—	2	3	6	10	2	4	5	2	3	1	2	—	—	—	—	—	—	1	—	—	43
8·125— 9·125	—	—	—	—	2	4	1	—	4	2	—	—	1	1	—	—	1	1	—	—	—	1	1	—	19
9·125—10·125	—	—	—	—	—	1	4	1	3	—	—	1	4	1	1	1	1	—	—	1	—	—	—	1	20
10·125—11·125	—	—	—	1	—	1	—	—	1	—	—	1	1	—	—	—	1	—	—	—	—	—	—	—	6
11·125—12·125	—	—	—	—	—	—	2	1	1	3	—	1	—	2	—	1	—	—	2	—	1	—	—	—	14
12·125—13·125	—	—	—	—	—	—	—	1	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	4
13·125—14·125	—	—	—	—	1	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	3
14·125—15·125	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	3
15·125—16·125	—	—	—	—	1	—	2	1	—	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	7
16·125—17·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17·125—18·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
18·125—19·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19·125—20·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20·125—21·125	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals ...	1	6	11	21	49	43	49	39	37	25	32	13	18	13	12	3	5	2	11	2	2	6	2	1	403

TABLE XI.

Correlation of Hearts and Spleens in Pneumonia. Males. (Cases of Pneumonia with Valvular Disease of Heart and Tubercular Pneumonia are excluded.)

Hearts in ozs.

Spleens in ozs.	6·125— 7·125	7·125— 8·125	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	.....	26·125—27·125	Totals
1·125— 2·125	—	—	1	—	1	1	—	—	—	—	—	—	—	—	...	—	3
2·125— 3·125	—	—	1	1	2	—	—	—	—	—	—	1	—	—	...	—	5
3·125— 4·125	1	—	1	4	4	1	3	4	2	1	—	—	—	—	...	—	21
4·125— 5·125	—	—	2	2	3	4	3	2	5	2	—	—	—	—	...	—	23
5·125— 6·125	—	1	—	5	6	9	8	4	2	3	1	—	1	1	...	1	42
6·125— 7·125	—	—	—	6	4	2	4	2	—	1	1	—	2	3	...	—	25
7·125— 8·125	—	—	—	2	5	4	3	2	—	2	—	1	—	1	...	—	19
8·125— 9·125	—	—	—	—	1	5	3	2	1	1	—	1	—	—	...	—	14
9·125—10·125	—	—	1	1	1	3	2	2	—	—	—	1	—	—	...	—	11
10·125—11·125	—	—	1	—	1	—	—	—	—	—	—	—	—	—	...	—	2
11·125—12·125	—	—	—	—	—	1	—	—	1	—	—	—	—	—	...	—	2
12·125—13·125	—	—	—	—	—	—	1	—	—	—	—	—	—	—	...	—	1
13·125—14·125	—	—	—	—	—	3	—	1	—	—	—	—	—	—	...	—	4
14·125—15·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
15·125—16·125	—	—	—	—	—	2	—	—	—	—	—	—	1	—	...	—	3
16·125—17·125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	...	—	1
17·125—18·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	...	—	1
Totals ...	1	1	7	21	28	35	27	20	12	10	2	3	4	5	...	1	177

TABLE XII.

Correlation of Hearts and Spleens in Valvular Disease of Heart and Aortic Aneurism. Males.

Hearts in ozs.

Spleens in ozs.

	8·125— 9·125	9·125—10·125	10·125—11·125	11·125—12·125	12·125—13·125	13·125—14·125	14·125—15·125	15·125—16·125	16·125—17·125	17·125—18·125	18·125—19·125	19·125—20·125	20·125—21·125	21·125—22·125	22·125—23·125	23·125—24·125	24·125—25·125	25·125—26·125	26·125—27·125	27·125—28·125	28·125—29·125	29·125—30·125	30·125—31·125	31·125—32·125	32·125—33·125	33·125—34·125	34·125—35·125	35·125—36·125	.....	44·125—45·125	Totals
1·125— 2·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
2·125— 3·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	...	—	1
3·125— 4·125	1	1	—	2	—	1	2	1	1	2	1	1	2	2	—	—	—	—	1	—	—	—	—	—	—	—	1	...	—	19
4·125— 5·125	1	—	2	—	1	2	1	1	—	2	1	3	2	1	1	1	1	—	1	—	—	—	—	—	1	—	—	...	—	22
5·125— 6·125	—	—	—	—	2	1	2	1	1	3	1	4	—	2	1	2	—	—	—	2	—	—	—	—	—	—	1	...	1	23
6·125— 7·125	—	—	—	1	2	1	1	3	—	—	—	3	—	2	—	2	2	1	1	—	—	—	1	—	—	—	—	...	1	21
7·125— 8·125	—	—	—	2	1	1	—	—	2	2	2	2	—	—	1	2	—	—	—	—	—	—	—	—	—	—	—	...	—	15
8·125— 9·125	—	—	1	2	—	—	3	—	1	2	—	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	...	—	11
9·125—10·125	—	—	—	—	1	—	—	2	1	4	—	1	—	2	—	1	—	—	—	—	1	1	—	—	—	1	—	...	—	15
10·125—11·125	—	—	—	—	—	1	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	4
11·125—12·125	—	1	—	1	—	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	1	—	—	—	...	—	5
12·125—13·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	...	—	3
13·125—14·125	—	—	—	1	—	—	—	2	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	6
14·125—15·125	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	...	—	4
15·125—16·125	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	3
16·125—17·125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
17·125—18·125	—	—	—	1	—	—	—	—	—	1	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	...	—	5
18·125—19·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	...	—	1
19·125—20·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
20·125—21·125	—	—	—	1	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	3
.....	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...
25·125—26·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
26·125—27·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
27·125—28·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
28·125—29·125	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
29·125—30·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	...	—	1
30·125—31·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
31·125—32·125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
Totals ...	2	2	3	11	7	8	9	12	11	20	7	18	6	9	4	10	4	3	4	3	2	2	1	2	1	2	—	2	...	1	166

SUI MASSIMI DELLE CURVE DIMORFICHE.

DAL DR FERNANDO DE HELGUERO, Roma.

SOMMARIO.

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1. *Introduzione.*

È ben nota la importanza che ha nella Biometrica la curva detta *normale* avente l' equazione in coordinate cartesiane ortogonali:

$$y = y_0 e^{-\frac{(x-b)^2}{2\sigma^2}}.$$

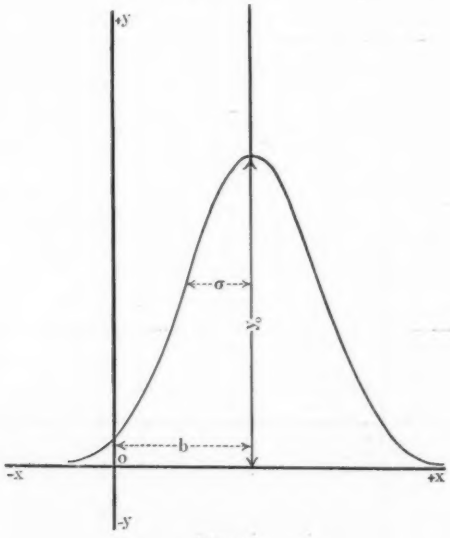


FIG. 1.

Essa ha tre parametri: y_0, b, σ, pei quali adotteremo le denominazioni di *ordinata massima* (y_0), *media* (b) e *deviazione normale* (standard deviation) (σ): le y_0 e σ sono sempre positive.

La sua forma è nota (Vedi fig. 1): in un piano cartesiano di assi $y=0$, $x=0$ essa giace intieramente dalla parte delle y positive, è simmetrica rispetto alla retta $x=b$ che dicesi *asse della curva normale.* La y è massima per $x=b$ ed ha il valore y_0, va da ambe le parti decrescendo continuamente fino a rendersi asintotica all' asse delle x, che dicesi *base della curva.* Esistono inoltre due flessi corrispondenti alle ascisse $b \pm \sigma$.

Si è trovato che questa curva serve ad esprimere le variazioni di molti caratteri in gruppi omogenei di organismi. Se invece si considera un materiale eterogeneo, risultante dalla mescolanza di due gruppi omogenei, si ottengon delle curve di frequenza dette *dimorfiche,* osservate per la prima volta dal Livi*. Queste posson graficamente ottenersi tracciando sulla stessa base due curve normali e su ogni

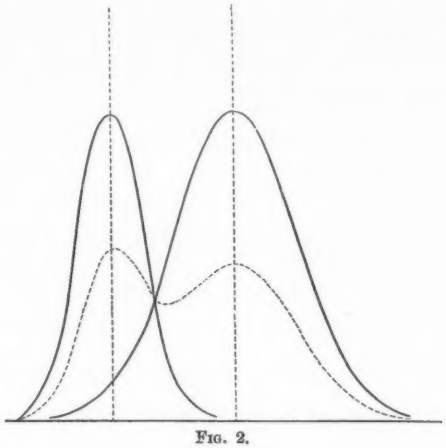


FIG. 2.

ordinata prendendo la somma delle frequenze corrispondenti alle due curve (Vedi fig. 2). La loro equazione è

$$y = y_1 e^{-\frac{(x-b_1)^2}{2\sigma_1^2}} + y_2 e^{-\frac{(x-b_2)^2}{2\sigma_2^2}},$$

se

$$y = y_1 e^{-\frac{(x-b_1)^2}{2\sigma_1^2}}, \quad y = y_2 e^{-\frac{(x-b_2)^2}{2\sigma_2^2}}$$

sono le equazioni delle curve componenti.

Riescono evidenti, per questo modo di generazione, le seguenti proprietà delle curve dimorfiche:

* R. Livi "Sulla statura degli Italiani," Firenze, 1883. Vedi anche *Archivio per l' Antropologia e l' Etnologia,* Vol. 13, Firenze, 1883; e *Annali di Statistica,* Vol. 8, 1883, pp. 119—156.

Come le curve normali giacciono intieramente nel semipiano delle y positive e restan sempre a distanza finita dall' asse delle x. Inoltre devon rendersi anch' esse asintotiche a questo asse: perciò devono ammettere dei massimi le cui ascisse diremo *mode*†. Il modo di formazione ci dice che questi saranno al più due, come verificheremo rigorosamente in seguito.

Possiamo avere, cioè, la *unimodalità* (un solo massimo), o la *bimodalità* (due massimi separati da un minimo); queste due forme erano già state osservate dal Livi. Caso limite fra i due è quello osservato dal Weldon‡ in cui la curva presenta un solo massimo, ed un flesso (a destra od a sinistra) con tangente parallela all' asse delle x: allora può pensarsi che il minimo della curva bimodale sia venuto a coincidere con uno dei massimi.

La unimodalità o bimodalità dipende evidentemente dai parametri delle curve componenti: in questo scritto noi ci proponiamo la seguente ricerca:

Dati i parametri vedere se la curva dimorfica è unimodale o bimodale.

Il numero dei massimi dipende solo da alcune funzioni dei parametri delle componenti; infatti è questo un carattere invariantivo per spostamenti paralleli dell' asse delle y e per cambiamento delle scale delle x e delle y. Potremo perciò prendere $b_1 = 0$, ed allora b_2 che indicheremo con b' ci rappresenterà la distanza fra gli assi. Inoltre prendendo opportunamente la scala delle y potremo fare $y_1 = 1$ ed y_2' sarà il rapporto delle ordinate massime. Della scala delle x disporremo in seguito.

La equazione della curva dimorfica è allora:

$$y = e^{-\frac{x^2}{2\sigma_1^2}} + y_2' e^{-\frac{(x-b')^2}{2\sigma_2^2}},$$

in cui supponiamo $\sigma_2^2 \leqslant \sigma_1^2$.

Per studiare i massimi e minimi di questa funzione dovremo cercare i valori della variabile x che annullano la $\frac{dy}{dx}$; abbiamo così:

$$0 = -\frac{x}{\sigma_1^2} e^{-\frac{x^2}{2\sigma_1^2}} - y_2' \frac{(x-b')}{\sigma_2^2} e^{-\frac{(x-b')^2}{2\sigma_2^2}};$$

dividendo per $\frac{e^{-\frac{x^2}{2\sigma_1^2}}}{\sigma_2^2}$

† Per la quasi assoluta mancanza di scritti di Biometrica in lingua italiana, ho dovuto tradurre in italiano il nome di questo parametro dall' inglese *the mode*, introdotto dal Prof. K. Pearson dal francese *le mode*. Ho adottato la parola *moda*, anzichè *modo* o *modulo* per seguire i francesi, perchè mi è sembrata meglio corrispondente al significato del parametro. "Since this value may be said figuratively to be the most fashionable one, it has been called *the mode*." C. B. Davenport "The Statistical Study of Evolution" in *The Popular Science Monthly*, Sept. 1901.

‡ Citato dal Pearson, *Phil. Trans.* Vol. 185, A. pp. 95 *et seq.* Vedi anche C. B. Davenport *Statistical Methods with special reference to Biological Variation*, New York, 1899, p. 28, fig. 26.

si ha: $$0 = x + y_2' \frac{\sigma_1^2}{\sigma_2^2}(x - b')\, e^{-\frac{x^2\left(1-\frac{\sigma_2^2}{\sigma_1^2}\right)}{2\sigma_2^2} + \frac{2xb'}{2\sigma_2^2} - \frac{b'^2}{2\sigma_2^2}}.$$

Poniamo: $$y_2' \frac{\sigma_1^2}{\sigma_2^2} = k, \quad 1 - \frac{\sigma_2^2}{\sigma_1^2} = h, \quad b = \frac{b'}{\sigma_2\sqrt{2}},$$

inoltre cambiamo la scala delle x prendendo la nuova $x = \frac{x}{\sigma_2\sqrt{2}}$; la nostra equazione diviene allora:

$$(a) \quad 0 = x + k(x - b)\, e^{-(hx^2 - 2bx + b^2)} = f(x),$$

e di questa dobbiamo studiare le radici. In essa figurano tre parametri k, h, b che sono espressi in funzione dei parametri delle curve componenti dalle relazioni:

$$k = \frac{y_2}{y_1}\frac{\sigma_1^2}{\sigma_2^2}, \quad h = 1 - \frac{\sigma_2^2}{\sigma_1^2}, \quad b = \frac{b_2 - b_1}{\sigma_2\sqrt{2}},$$

e supponendo $\sigma_2 \leqslant \sigma_1$, $b_2 \geqslant b_1$ (il chè non toglie generalità alla ricerca perchè se fosse $b_2 < b_1$ potremmo invertire il senso dell' asse delle x), i parametri k, h, b devon sempre soddisfare alle diseguaglianze:

$$k > 0, \quad 0 \leqslant h < 1, \quad b \geqslant 0.$$

2. *Criterio per giudicare del numero dei massimi delle curve dimorfiche.*

Noi studieremo adesso le radici della $f(x) = 0$ per risolvere la questione prima enunciata:

Dati i parametri delle curve componenti, e perciò k, h, b, vedere quanti massimi ha la curva dimorfica. Dovremo perciò cercare quando la equazione (a) ammette una sola radice reale e quando ne ammette 3.

Nel caso particolare $b = 0$ (ovvero $b_1 = b_2$), cioè curve componenti con assi coincidenti, la $f(x) = 0$ si spezza nelle due $x = 0$, ed $1 + ke^{-hx^2} = 0$ che non ammette radici reali perchè k e l' esponenziale sono sempre positivi; onde in questo caso esiste un solo massimo, per $x = 0$, corrispondente al massimo comune delle curve componenti, e ciò era evidente.

L' altro caso particolare $h = 0$, onde $\sigma_1 = \sigma_2$, lo considereremo a parte. Nel caso generale in cui

$$k > 0, \quad 0 < h < 1, \quad b > 0$$

ci ricondurremo allo studio delle radici di una equazione algebrica di terzo grado.

In generale la (a) può scriversi:

$$0 = \frac{x}{k(x - b)} + e^{-(hx^2 - 2bx + b^2)}.$$

Ponendo: $$(b) \quad z_1 = e^{-(hx^2-2bx+b^2)},$$

$$(c) \quad z_2 = \frac{x}{k(b-x)},$$

per le radici della (a) avremo $z_1 = z_2$.

Studiamo le due curve (b) e (c); se le riferiamo agli stessi assi, le ascisse delle loro intersezioni ci daranno le radici della (a).

Studio di (b).

$$z_1 = e^{-(hx^2-2bx+b^2)}.$$

Essa è una curva *normale*. Infatti la sua equazione può scriversi:

$$z_1 = e^{b^2\left(\frac{1-h}{h}\right)} \cdot e^{-\frac{\left(x-\frac{b}{h}\right)^2}{\frac{1}{h}}}.$$

La moda è $\frac{b}{h}$, la deviazione normale $\frac{1}{\sqrt{2h}}$, l'ordinata massima $e^{b^2\frac{1-h}{h}}$.

Le sue proprietà sono ben note, e perciò è inutile insistervi.

Studio di (c).

$$z_2 = \frac{x}{k(b-x)}.$$

Può scriversi $$kz_2x - kbz_2 + x = 0.$$

È una *iperbole equilatera* che passa per l' origine; asintoti sono le rette

$$x = b, \quad z_2 = -\frac{1}{k}.$$

Per $x < 0$ o per $x > b$, z_2 è negativo; onde per studiare le intersezioni colla (b) che ha l' ordinata sempre positiva, basta considerare il ramo per $0 < x < b$; in esso la z_2 cresce con continuità da 0 ad ∞ $\left(\text{la } \frac{dz_2}{dx} = \frac{b}{k(b-x)^2} \text{ è sempre positiva}\right)$.

Da queste prime osservazioni possiam dedurre che le radici della (a), di cui una almeno è sempre reale, sono comprese nell' intervallo $(0, b)$. In questo intervallo tanto la (b) che la (c) son sempre crescenti, positive. (Infatti il massimo della (b) è per $x = \frac{b}{h} > b$). Vedi fig. 3.

Per continuare il nostro studio formiamo la derivata della $f(x)$: essa è

$$\frac{df(x)}{dx} = 1 + k\{1 - (x-b)(2hx-2b)\}e^{-(hx^2-2bx+b^2)},$$

uguagliandola a zero:

$$\frac{1}{k(2hx^2 - 2b(1+h)x + 2b^2 - 1)} = e^{-(hx^2-2bx+b^2)}.$$

Ponendo: $(b)\quad z_1 = e^{-(hx^2-2bx+b^2)}$,

$$(d)\quad z_3 = \frac{1}{k\{2hx^2 - 2b(1+h)x + 2b^2 - 1\}},$$

alle radici della $\frac{df(x)}{dx} = 0$ corrispondono le ascisse delle intersezioni delle due curve (b) e (d) quando siano inferite agli stessi assi.

La (b) è già stata veduta.

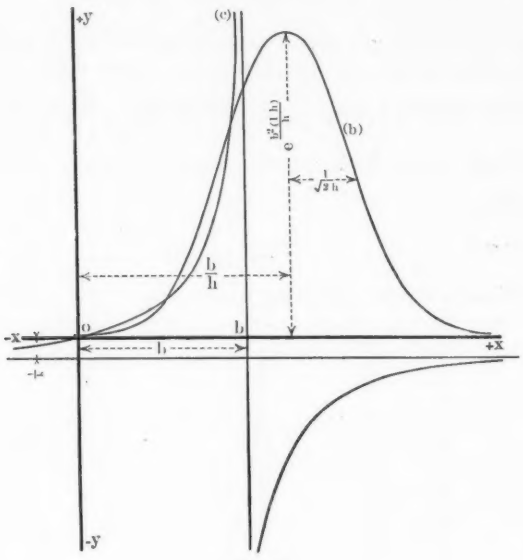


FIG. 3.

Studio di (d).

$$z_3 = \frac{1}{k\{2hx^2 - 2b(1+h)x + 2b^2 - 1\}}.$$

Per x finito z_3 non si annulla mai, però l' asse delle x è un asintoto della curva. Perchè z_3 cambi segno deve passare per il valore ∞ e ciò avviene in corrispondenza alle radici della:

$$2hx^2 - 2b(h+1)x + 2b^2 - 1 = 0,$$

che sono $\quad \begin{matrix} x_1 \\ x_2 \end{matrix} \Big\{ = \frac{b(h+1) \mp \sqrt{b^2(h^2+2h+1) - 2h(2b^2-1)}}{2h}.$

ovvero: $\quad \begin{matrix} x_1 \\ x_2 \end{matrix} \Big\{ = \frac{b(h+1) \mp \sqrt{b^2(1-h)^2 + 2h}}{2h}.$

Ora se $1-2b^2$ è negativo, le due radici sono entrambe positive, se $1-2b^2$ è positivo, è positiva solo la x_2. Io dico che in ogni caso x_2 è maggiore di b; infatti si ha:

$$x_2 = \frac{b(h+1)+\sqrt{b^2(1-h)^2+2h}}{2h} > \frac{b(h+1)+b(1-h)}{2h} = \frac{b}{h} > b.$$

Riguardo ad x_1, è positivo solo se $2b^2 > 1$, $b > \frac{1}{\sqrt{2}}$; in questo caso è minore di

$$\frac{b(h+1)}{2h}.$$

In ogni caso z_3 è positivo per $x < x_1$, negativo per $x_1 < x < x_2$, positivo di nuovo per $x > x_2$. Poichè a noi la curva interessa solo nel tratto $0 < x < b$, segue che per $b < \frac{1}{\sqrt{2}}$, nel nostro tratto z_3 è sempre negativo, mentre è almeno in parte positivo se $b > \frac{1}{\sqrt{2}}$. È facile vedere che in questo intervallo è sempre crescente, infatti la derivata è positiva.

Inoltre per $x = 0$, $\qquad z_3 = \frac{1}{k(2b^2-1)}$.

La curva è tracciata in figura. (Vedi fig. 4.)

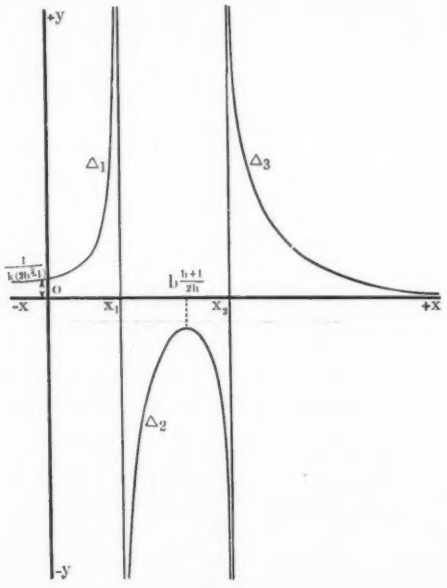


Fig. 4.

A noi interessa solo il ramo indicato colla lettera Δ_1 che partendo dal punto $\left(x=0,\ z_3=\frac{1}{k(2b^2-1)}\right)$ cresce fino a divenire asintotico alla retta $x=x_1$. Consideriamo ora le intersezioni delle due curve (c) e (d) nell' intervallo $0<x<b$; le loro ascisse sono le radici della equazione di terzo grado:

$$\frac{x}{b-x}=\frac{1}{\{2hx^2-2b(h+1)x+2b^2-1\}},$$

cioè, in generale,

$$2hx^3-2b(h+1)x^2+2b^2x-b=0.$$

La indicheremo con $R(x)=0$. Si ha:

$$R(-\infty)<0,$$
$$R(0)=-b,$$
$$R(b)=-b,$$
$$R(+\infty)>0.$$

Perciò la $R(x)=0$ ammette certo una radice reale, positiva, maggiore di b, e nell' intervallo $(0, b)$ o due o nessuna.

Ciò posto supponiamo che la $f(x)=0$ ammetta tre radici reali e distinte x', x'', x''', con $x'<x''<x'''$. Ad esse corrisponderanno 3 intersezioni delle curve (b) e (c), indicate nella fig. 5 colle lettere A, B, C. Per ciò che abbiamo visto studiando queste curve sarà anche

$$0<x'<x''<x'''<b.$$

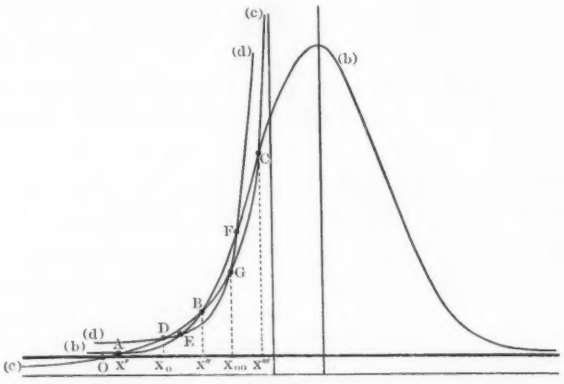


Fig. 5.

Poichè la $f(x)$ si annulla in x' ed x'', per la continuità, la $\frac{df(x)}{dx}=0$ fra x' ed x'' deve ammettere una radice (od un numero dispari di radici); così fra x'' ed x'''. Perciò la (d) deve tagliare la curva normale (b) in almeno un punto E compreso fra A e B, ed in F fra B e C.

Io dico che la (d), nell' intervallo $x=0$, $x=b$, taglierà la (c) in due punti D, G compresi rispettivamente fra A e B, B e C, ed in essi soltanto.

Infatti in più di due punti non può tagliarla perchè se no l' equazione $R(x)=0$, che si ha considerando le intersezioni delle (c) e (d), avrebbe più di due radici reali fra 0 e b contro ciò che abbiam visto; inoltre la (c) essendo continua, monodroma, continuamente crescente, deve tagliare il contorno chiuso $AEBD$ in un numero pari di punti: ma essa taglia in un punto, od in un numero dispari di punti il lato AEB, onde dovrà tagliare in un punto D anche l' altro lato ADB.

Nello stesso modo si dimostra la seconda intersezione in G. Siano x_0, x_∞ le ascisse corrispondenti a queste due intersezioni: saranno le radici di $R(x)=0$ comprese fra 0 e b. Segue che:

Condizione necessaria perchè la curva sia bimodale è che la $R(x)=0$ *abbia due radici reali* x_0, x_∞, *fra* 0 *e* b, *e che per esse la successione dei segni di* $f(0)$, $f(x_0)$, $f(x_\infty)$ *abbia due variazioni.* La seconda parte è giustificata dal fatto che

$$x' < x_0 < x'', \quad x'' < x_\infty < x'''$$

perchè nell' intervallo $(0, b)$ le tre curve (b), (c), (d) son sempre crescenti: così in ciascuno degli intervalli $(0, x_0)$, (x_0, x_∞) cade una sola radice di $f(x)=0$. Notiamo che $f(0)=-kbe^{-b^2}$ è sempre negativo.

Questa condizione è anche sufficiente; infatti se essa è soddisfatta, $f(x)$, per la continuità, ammette una radice fra 0 ed x_0, una seconda fra x_0 ed x_∞, una terza fra x_∞ e b, poichè allora l' unica possibile successione di segni è:

$$\begin{array}{cccc} f(0), & f(x_0), & f(x_\infty), & f(b)=b. \\ - & + & - & + \end{array}$$

Questo procedimento mostra anche che le curve dimorfiche non possono avere più di due massimi, poichè la $R(x)=0$ non può ammettere più di due radici reali fra 0 e b.

Il caso limite fra la unimodalità e la bimodalità, cioè la curva con un flesso con tangente orizzontale si ha quando $f(x_0)=0$ ovvero $f(x_\infty)=0$.

Infatti allora deve aversi per lo stesso valore di x, corrispondente a due delle radici x', x'', x''' che vengono a coincidere:

$$f(x)=0, \quad \frac{df(x)}{dx}=0,$$

cioè nel punto corrispondente a questa ascissa la (b) deve tagliare tanto la (c) che la (d), perciò questo x dev' esser radice di $R(x)=0$: uno dei due numeri x_0, x_∞.

Si hanno due condizioni per il caso limite ($f(x_0)=0$ ovvero $f(x_\infty)=0$) perchè nella curva dimorfica il flesso con tangente orizzontale può trovarsi a destra (Vedi fig. 6) od a sinistra del massimo, e nella figura 5 perchè posson venire a coincidere i punti A e B, od i B e C.

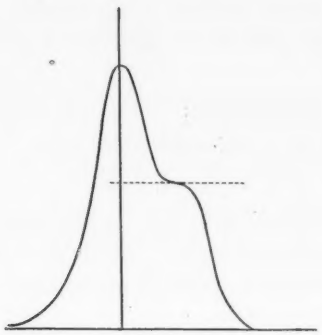


FIG. 6.

Riepilogando, il procedimento per vedere se la curva dimorfica, di cui son dati i 6 parametri y_1, y_2, b_1, b_2, σ_1, σ_2, ha uno o due massimi è il seguente:

(1) Si costruiscono le funzioni dei parametri k, h, b.

(2) Si cercano le radici reali di

$$R(x) = 2hx^3 - 2b(h+1)x^2 + 2b^2x - b = 0$$

(che può anche scriversi $2x(x-b)(hx-b) - b = 0$)

comprese fra 0 e b. Se non ne esistono o ve n' è una doppia la curva è

.....*unimodale.*

Se ci sono due radici reali e distinte x_0, x_∞:

(3) Si sostituiscono in $f(x)$. Possono allora darsi i seguenti casi

o $f(x_0) = 0$ o $f(x_\infty) = 0$ e si ha il *caso limite,*

o $f(x_0) < 0$ e la curva è *unimodale,*

o $f(x_0) > 0$, e in tal caso se $f(x_\infty) > 0$ curva *unimodale,*

se $f(x_\infty) < 0$ curva *bimodale.*

Così la ricerca è completamente risoluta nel caso generale.

Riguardo al valore delle mode x', x''' nel caso della bimodalità si attengono per approssimazione come radici della $f(x) = 0$: esse sono già separate poichè

$$0 < x' < x_0 \quad \text{e} \quad x_\infty < x''' < b.$$

Si ricordi che si è effettuata la trasformazione $x = \dfrac{x}{\sigma_2\sqrt{2}}$.

3. *Condizioni sufficienti per la unimodalità.*

Un caso in cui si può asserire subito la unimodalità è quello in cui $b < \dfrac{1}{\sqrt{2}}$; allora abbiamo visto che la (d) giace per $0 < x < b$ dalla parte delle ordinate

negative, perciò non può incontrare la (*c*) che in quell' intervallo è sempre dalla parte positiva, e la $R(x)=0$ che dà le ascisse delle intersezioni delle (*c*) e (*d*), non ha tra 0 e *b* radici reali.

Onde ricordando le posizioni fatte:

Condizione sufficiente per la unimodalità è:

$$b_2 - b_1 < \sigma_2.$$

Cioè: La curva dimorfica è unimodale se la distanza tra gli assi delle componenti è minore della più piccola delle deviazioni normali.

Ricordiamo che la deviazione normale è geometricamente la distanza dei flessi dall' asse.

Io dico che la curva è certo unimodale anche se la distanza tra gli assi è minore del doppio di questa quantità.

Per stabilire questo risultato bisogna studiare la equazione $R(x)=0$ che può scriversi sotto le due forme identiche:

$$2hx^3 - 2b(h+1)x^2 + 2b^2x - b = 0,$$

$$2x(x-b)(hx-b) - b = 0.$$

Osserviamo la curva $y=R(x)$: essa incontra la retta $y=-b$ nei punti

$$x_1 = 0, \quad x_2 = b, \quad x_3 = \frac{b}{h}.$$

e poichè $0<h<1$ si ha $x_1 < x_2 < x_3$. A noi interessano le radici comprese fra 0 e *b* e perciò solo in questo tratto vogliamo studiare la $y=R(x)$, così non teniamo conto di x_3.

Considerando la derivata

$$\frac{dR(x)}{dx} = 6hx^2 - 4b(h+1)x + 2b^2 = 2S(x),$$

si vede che i punti di massimo o minimo x_0', x_0'', sono compresi negli intervalli $(0, b)$; $(b, +\infty)$ come ci indica la successione dei segni di $S(0)=+b^2$, $S(b)<0$, $S(+\infty)=+\infty$.

Onde nell' intervallo $(0, b)$ c' è un solo punto x_0', di massimo (infatti ivi la $\frac{d^2R(x)}{dx^2} = 4\{3hx - b(h+1)\}$ è negativa). La figura 7 mostra l' andamento della curva in questo tratto.

Il discriminante di $R(x)=0$ è $R(x_0')$, poichè secondo che

$$R(x_0') \begin{matrix} < \\ = \\ > \end{matrix} 0 \begin{matrix} \text{non esistono radici reali} \\ \text{esiste una radice doppia} \\ \text{esistono due radici reali e distinte} \end{matrix}$$

di $R(x)=0$ comprese fra 0 e *b*.

Il valore di x_0' è

$$x_0' = \frac{b\{h+1-\sqrt{(h+1)^2-3h}\}}{3h}.$$

Sostituendo e sviluppando si trova che le diseguaglianze $R(x_0') \lessgtr 0$ equivalgono alle

$$\frac{\{9h(h+1)-2(h+1)^3\}+2\{(1+h)^2-3h\}^{\frac{3}{2}}}{27h^2} \lessgtr \frac{1}{2b^2}.$$

Vogliamo ora vedere il massimo valore di b indipendente da h e da k per il quale si può asserire la unimodalità della curva.

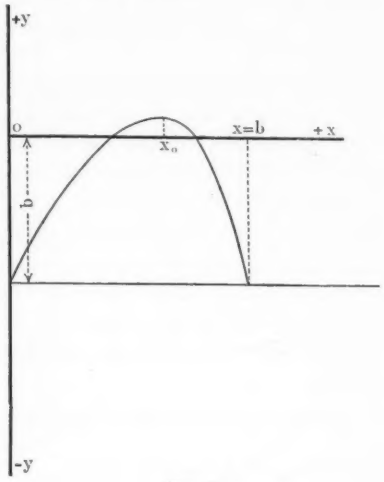


FIG. 7.

Perciò scriviamo la condizione sotto la forma:

$$\{(h+1)^2-3h\}^3 < \{(h+1)^3+h(\gamma h-\tfrac{9}{2})\}^2,$$

dove $$\gamma = \frac{27}{4b^2} - \frac{9}{2}.$$

Sviluppando si trova:

$$\gamma^2h^2+\gamma(2h^3+6h^2-3h+2)+9h^3+\tfrac{9}{4}>0.$$

Cerchiamo il minimo valore di γ per cui questa è soddisfatta qualunque sia h fra 0 ed 1.

Prendendo h molto piccolo si ha:

$$2\gamma+\tfrac{9}{4}>0, \quad \gamma>-\tfrac{9}{8}.$$

Anche per il valore $\gamma=-\frac{9}{8}$ la diseguaglianza è realmente soddisfatta; infatti sostituendo e riducendo si ha

$$h^2-\tfrac{13}{16}h+\tfrac{1}{2}>0,$$

che è verificata per $h=0$, per $h=1$ ed anche per $h=\frac{13}{32}$ per cui il primo membro assume il minimo valore: $\frac{1}{2}\{1-\frac{1}{2}(\frac{13}{16})^2\}>0$.

Ricordando la posizione $\gamma=\frac{9}{2}\left(\frac{3}{2b^2}-1\right)$, la condizione $\gamma>-\frac{3}{8}$ equivale a $b^2<2$.

Perciò la curva è certo unimodale se

$$b_2-b_1<2\sigma_2:$$

Come avevamo prima affermato.

Potrebbero trovarsi altre condizioni per la unimodalità pensando b non costante, caso completamente trattato, ma funzione di h. Con ciò si verrebbero a stabilire delle relazioni fra b_2-b_1, σ_2 e σ_1. Però non si ottengono così condizioni notevolmente più estese di quelle già poste.

Si potrebbe trovare in modo analogo un numero b_0 tale che per $b>b_0$ il discriminante sia positivo qualunque sia h, però *non si possono stabilire delle condizioni sufficienti per la bimodalità indipendenti da k.* Infatti sappiamo che condizione necessaria per la bimodalità è che $f(x_0)$ ed $f(x_\infty)$ siano il primo positivo, il secondo negativo. Possiamo scrivere:

$$f(x_0)=A_1-kB_1, \qquad f(x_\infty)=A_2-kB_2,$$

dove A_1B_1, A_2B_2 sono quantità positive. Se esse sono indipendenti da k, io posso sempre imaginare k così piccolo o così grande che siano $f(x_0)$ ed $f(x_\infty)$ entrambi positivi o negativi. Perciò non posson trovarsi valori di b costanti, o funzioni di h indipendenti da k, per i quali la curva dimorfica sia necessariamente bimodale.

4. *Caso particolare: curve componenti di ugual deviazione normale:* $h=0$.

In questo caso uguagliando a zero la derivata si ha:

$$x+k(x-b)\,e^{2bx-b^2}=0.$$

Poniamo $2bx-b^2=z$ da cui $x=\frac{z+b^2}{2b}$ e $x-b=\frac{z-b^2}{2b}$.

L' equazione diviene:

$$z+b^2+k(z-b^2)\,e^z=0=F(z).$$

Dobbiam vedere quando questa ammette uno e quando tre radici reali: a queste corrispondono in generale le ascisse delle intersezioni delle due curve:

$$z_1=-\frac{z+b^2}{z-b^2}, \qquad z_2=ke^z.$$

La
$$\frac{dF(z)}{dz}=0=1+k\{z-b^2+1\}\,e^z$$

ha per radici le ascisse delle intersezioni delle curve:

$$z_3=-\frac{1}{z-b^2+1}, \qquad z_2=ke^z.$$

Una di queste, la z_2, è sempre crescente, sempre positiva: Per z crescente da $-\infty$ a 0, z_2 cresce da 0 a k, per z crescente da 0 a $+\infty$ la z_2 varia da k ad ∞.

Le altre due curve sono iperbole equilatere.

Per la prima si ha che per

$$z > b^2, \quad z_1 < 0,$$
$$-b^2 < z < b^2, \quad z_1 > 0,$$
$$z < -b^2, \quad z_1 < 0;$$

per la seconda per

$$z > b^2 - 1, \quad z_3 < 0,$$
$$z < b^2 - 1, \quad z_3 > 0.$$

Le intersezioni della z_1 colla z_2 sono comprese nella striscia limitata dalle rette $z = -b^2$ e $z = +b^2$.

Quelle delle z_3 colla z_2 nell' intervallo $z = -\infty$ e $z = -(1-b^2)$. Supponiamo che la $F(z) = 0$ ammetta tre radici reali e distinte: allora fra due intersezioni della z_1 colla z_2, deve esserci una intersezione della z_3 colla z_2. (Vedi fig. 8.)

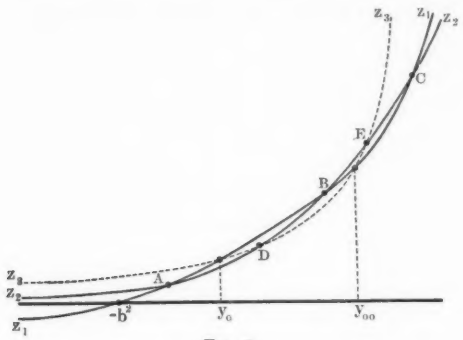


FIG. 8.

Siccome son tutte curve omali crescenti, le intersezioni delle z_1 e z_2 devon comprendere quelle delle z_1 e z_3. Le ascisse di queste siano y_0, y_∞. Esse sono le radici della:

$$(z+b^2)(z-b^2)+(z+b^2) = z-b^2,$$

ovvero $$z^2 - b^4 + 2b^2 = 0 \text{ onde } y_0 = -b\sqrt{b^2-2},$$

$$y_\infty = +b\sqrt{b^2-2}.$$

Inoltre le $F(-b^2)$, $F(y_0)$, $F(y_\infty)$, $F(b^2)$ devon presentare segni alternati.

Ora $$F(-b^2) < 0 \text{ e } F(+b^2) > 0,$$

perciò devon essere $F(y_0)$ positivo ed $F(y_\infty)$ negativo.

Evidentemente queste condizioni sono anche sufficienti perchè la $F(z)=0$ ammetta tre radici reali e distinte.

Segue che: *Condizioni necessarie ed insieme sufficienti perchè la curva dimorfica risultante di componenti normali di ugual deviazione normale sia bimodale sono:*

(1) y_0 ed y_∞ reali onde $b > \sqrt{2}$,

(2) $\{b - \sqrt{b^2-2}\} - k\{b + \sqrt{b^2-2}\}\, e^{-b\sqrt{b^2-2}} > 0$

e $\{b + \sqrt{b^2-2}\} - k\{b - \sqrt{b^2-2}\}\, e^{b\sqrt{b^2-2}} < 0.$

Intanto *possiamo asserire la unimodalità se* $b < \sqrt{2}$, ovvero $b_2 - b_1 < 2\sigma$.

Se $b_2 - b_1 > 2\sigma$, allora dovremo vedere se sono soddisfatte entrambe le diseguaglianze prima scritte.

Nel caso che lo siano si ha la bimodalità; se anche una sola non è verificata la curva è unimodale.

Si ha il caso limite quando è soddisfatta una delle due relazioni:

$$\{b - \sqrt{b^2-2}\} - k\{b + \sqrt{b^2-2}\}\, e^{-b\sqrt{b^2-2}} = 0,$$

$$\{b + \sqrt{b^2-2}\} - k\{b - \sqrt{b^2-2}\}\, e^{b\sqrt{b^2-2}} = 0.$$

MISCELLANEA.

I. On some Dangers of Extrapolation.

By EMILY PERRIN, University College, London.

(1) It is well known that most excellent graduation results both in the case of physical and vital investigations may be obtained by fitting algebraic or trigonometrical curves to a limited range of observation. Such curves are in constant use for interpolation, not only in physics and demography, but also in chemistry, in actuarial science, and in empirical investigations in all branches of technical research. So long as such curves or algebraic formulae are applied for the purposes of interpolation within the range of observation, their use is not only justifiable, but indeed indispensable. This statement of course supposes that a good formula has been selected and has been properly fitted to the observations.

When we examine, however, the excellence of fit of even an arbitrarily chosen curve to a given range of data, when we see how it smooths the irregularities, and provides well within the limits of probable error the graduation that the chemist or engineer or actuary is eagerly seeking, we are tempted to continue our empirical representation of the data beyond the range of actual observation and predict the future from the past, or the condition of the unobserved from the observed. This extrapolation is one of the most fascinating fields of enquiry and yet one in which definite conclusions are most difficult to reach and pitfalls frequent and dangerous. How often is this extrapolation indulged in almost unconsciously by the scientific enquirer! We know now that the proportionality of stress and strain is only true within narrow limits, yet the early investigators extrapolated from this linearity all across the mysteries of set, yield-point, and stricture, up to rupture! Within quite recent years we have, we fancy, seen distinguished physicists and chemists running along tangents as if every observation-curve beyond the observed range ended in an indefinitely extended point of inflexion. Yet their want of logic is only comparable with that of the early physicists who extended Hooke's Law or Boyle's Law beyond the limits of observation, or of the later physicists who apply the deductions from Hooke's Law to the stress and strain of the solid earth, or relations between physical attributes at known temperatures and pressures to what may be supposed to happen beyond the range at which it has hitherto been possible to experiment.

The present enquiry is, of course, of a much humbler nature, but it may serve possibly as a warning on the dangers of extrapolation. Even if it does not deal with such important problems as the physics of the solid earth, it does deal with things which have been subjected to a like treatment. A parabola of a sufficiently high order—such a curve as has hitherto been often used by physicists and chemists—has been selected to describe the observed range of measurements, and then extrapolation has been attempted at a very small distance outside that range and the results compared with actual observation. When the conclusion to be drawn is merely a note of warning, may we not learn from the lesser as to the greater? It seems to us that such "extrapolation" at least is justifiable.

(2) The data dealt with are drawn from the demographic statistics of the City of Buénos-Ayres and are published in the *Annuaire Statistique de la Ville de Buénos-Ayres* of which twelve volumes have now appeared. This publication is to some extent a model for municipal statistical work, and we have to cordially thank M. Albert B. Martinez the *Directeur de la Statistique municipale* of Buénos-Ayres for our copy of this interesting book. Buénos-Ayres is a town which is altering demographically in two very sensible ways; there is a rapidly increasing population, together with improving sanitary and medical conditions, indicated by a falling death-rate and a decrease in the percentage of still-born children. The data in our possession started from the year 1887, but to allow of extrapolation beyond the start of the range we did not use that year. The following thirteen years' returns were then available for investigation at the time of the inquiry:

TABLE I.

City of Buénos-Ayres.

Year	Population on Dec. 31	Deaths per 1000	Still-born per 1000
1888	455,167	27·17	2·45
1889	523,452	28·15	2·49
1890	547,144	30·00	2·36
1891	535,060	24·32	2·46
1892	554,713	24·05	2·29
1893	580,371	22·40	2·22
1894	603,012	22·72	2·06
1895	677,780	22·05	1·73
1896	712,095	19·16	1·78
1897	738,484	19·25	1·78
1898	765,744	17·67	1·70
1899	795,323	17·06	1·63
1900	821,293	20·09*	1·68

The problem was then this: Graduate these results by a sufficiently high parabola, and predict by extrapolation the demographic condition of the town before and after the period covered by the above data.

The method of solution adopted was as follows: The successive moments of the observations about the middle point of the range were calculated by the processes indicated in *Biometrika*, Vol. II. pp. 15 *et seq.*, and the series of best fitting parabolas was determined. We halted when a thoroughly good fit had been found. Thus we stopped at the fifth order parabola in the case of population and at the fourth in the case of the death and still-born rates. Judged by any test of goodness of fit it will be found that *within the range dealt with* our curves give excellent graduation. For example, the mean error made in fitting the population with a straight line is 16·2 thousands, while the mean error made in using a curve of the fifth order is less than half this, being 6·5 thousands. This connotes a mean error in the population as estimated by interpolation of almost exactly one per cent., which may be considered for all purposes of interpolation in vital statistics as very satisfactory.

If we take the death-rate statistics we do not get anything like the same improvement if we pass from a straight line to a parabola of the fourth order. In fact the mean error made in the death-rate by the latter curve is 1·07, while it is only 1·18 if we use the best fitting straight line, or the fourth order parabola makes an error of 4·7 per cent. on an average in the death-rate, the straight line an error of 5·7 per cent. This is a sensible but not a considerable improvement, and it did not seem that a substantially greater gain would be made by continuing the process further. Clearly, when we consider the effect of epidemic sicknesses, we can hardly hope to get a closer result from a graduating curve.

* Small-pox epidemic.

Lastly, turning to the still-born rate, we have for the parabola of the fourth order a mean error of ·051 and for the straight line a mean error of ·075, corresponding to 2·5 and 3·7 percentage errors respectively. This is a sensible betterment, but nothing like as good as in the case of the population data. We may say that by using a high parabola instead of a straight line we better the fit two-and-a-half times for the population, one-and-a-half times for the still-born, and one-and-a-quarter times for the death-rate. In the last two cases therefore straight lines would give quite reasonable graduation or interpolation formulae.

The actual equations to the curves, the year 1894 being in each case taken as origin of x, the unit of which is a year, are:

Population. Straight line:

$$y = 639{\cdot}1667 + 29{\cdot}8186x.$$

Fifth order parabola:

$$y = 617{\cdot}729 + 44{\cdot}8543x + 3{\cdot}599{,}110x^2 - 1{\cdot}555{,}483x^3 - {\cdot}083{,}917x^4 + {\cdot}033{,}424x^5.$$

The unit of y is here 1000 individuals.

Death-rate per 1000. Straight line:

$$y = 22{\cdot}5383 - 1{\cdot}0428x.$$

Fourth order parabola:

$$y = 21{\cdot}9981 - 1{\cdot}345{,}685x + {\cdot}052{,}220x^2 + {\cdot}014{,}016x^3 - {\cdot}000{,}330x^4.$$

The unit of y is one in the thousand.

Still-born per 1000. Straight line:

$$y = 2{\cdot}0471 - {\cdot}08799x.$$

Fourth order parabola:

$$y = 2{\cdot}0173 - {\cdot}130{,}823x + {\cdot}005{,}316x^2 + {\cdot}001{,}983x^3 - {\cdot}000{,}131x^4.$$

The unit of y is one in the thousand.

The results obtained by observation and from calculation by the parabolas between the years 1888 and 1900 are given in the accompanying Table II. This table shows at once that the results are quite satisfactory within the range of observation. The same point is brought out graphically from the diagram. Confining our attention to the curves between the verticals Y_1Y_1 and Y_2Y_2, we could hardly hope to better the graduation provided.

TABLE II.

Interpolation Calculations.

Year	Population in 1000's		Death-Rate per 1000		Still-born Rate per 1000	
	Calculated	Difference	Calculated	Difference	Calculated	Difference
1888	446	− 9	28·50	+1·33	2·40	− ·05
1889	521	− 2	28·08	− ·07	2·47	− ·02
1890	540	− 7	27·24	−2·76	2·47	+ ·11
1891	543	+ 8	26·10	+1·78	2·39	− ·07
1892	552	− 3	24·78	+ ·73	2·28	− ·01
1893	578	− 2	23·38	+ ·98	2·15	− ·07
1894	618	+15	22·00	− ·72	2·02	− ·04
1895	665	−13	20·72	−1·28	1·89	+ ·16
1896	709	− 3	19·62	+ ·46	1·79	+ ·01
1897	744	+ 6	18·78	− ·47	1·72	− ·06
1898	768	+ 2	18·26	+ ·59	1·67	− ·03
1899	790	− 5	18·12	+1·06	1·66	+ ·03
1900	831	+10	18·40	−1·69	1·68	·00

But now let us look at the problem of extrapolation. Can we prophesy what happened in 1887, the year before our range of observation starts, or in the years 1901 and 1902, the data of which have been received since our curves were determined? Table III. gives the actually observed and the predicted results, (i) on the basis of a straight line extrapolation, (ii) on the basis of a high parabolic extrapolation.

TABLE III.

Extrapolation Calculations.

Year	Population in 1000's					Death-Rate per 1000					Still-born Rate per 1000				
	Parabola		Line		Actual	Parabola		Line		Actual	Parabola		Line		Actual
		Δ		Δ			Δ		Δ			Δ		Δ	
1887	251	−187	430	−·08	438	28·38	+ ·79	29·84	+2·25	27·59	2·20	+·36	2·66	+·82	1·84
1901	935	+ 87	848	·00	848	19·15	+ ·55	15·24	−3·36	18·60	1·73	+·09	1·43	−·21	1·64
1902	1162	+284	878	+·08	870	20·39	+4·19	14·20	−2·00	16·20	1·79	+·11	1·34	−·34	1·68
Mean Δ Extrapolation	—	186	—	·05	—	—	1·84	—	2·54	—	—	·19	—	·46	—
Mean Δ Interpolation	—	6·5	—	16·2	—	—	1·07	—	1·18	—	—	·05	—	·08	—

Now these results show that for the death-rate and the still-born rate the parabolas give better results for extrapolation than the straight lines, but that both representations have a mean error much in excess of the average mean error of interpolation within the range on which the calculation is based. An examination of the diagram shows that both rate curves are actually inadmissible a little beyond the range, for we see that either calculated rate tends to *fall* beyond the beginning of the range and *rise* after the end of it; there can hardly be a doubt that the very reverse of this must represent the actual state of affairs. In the case of the population curves this divergence from the facts does not occur; the total population falls before the beginning of the range and rises after the end of it as we should anticipate. But we see that the great closeness of the calculated to the observed population *within* the range has been gained by immensely emphasising the fall before and the rise after the range, so that the curve becomes worthless for purposes of extrapolation. Indeed while the parabola within the range is two to three times as good as the best representative line, the latter is indefinitely better than the former for extrapolation, giving indeed extremely good results. We are thus forced to the conclusions that:

(i) Empirical formulae which fit extremely well within the range of observation may give very bad extrapolation results only just outside that range.

(ii) Of two curves that which gives by far the better interpolation results, may give by far the worse extrapolation results. We cannot argue from excellency of fit within a given range to a fitness for prophesying what occurs even just outside that range.

Generally we believe that while empirical formulae—including in that term even general laws like Hooke's or Boyle's—may be excellent for interpolation within the range of actual observation, they cannot be used in either physics or vital statistics without extreme caution, if indeed they can be used at all, to predict what will occur even just outside that range. While interpolation and graduation can be satisfactorily carried out by the use of a variety of empirical curves, there is no corresponding method of extrapolation unless we have some solid reasons for assuming that the phenomena can only be represented by a formula or curve of a very definite type.

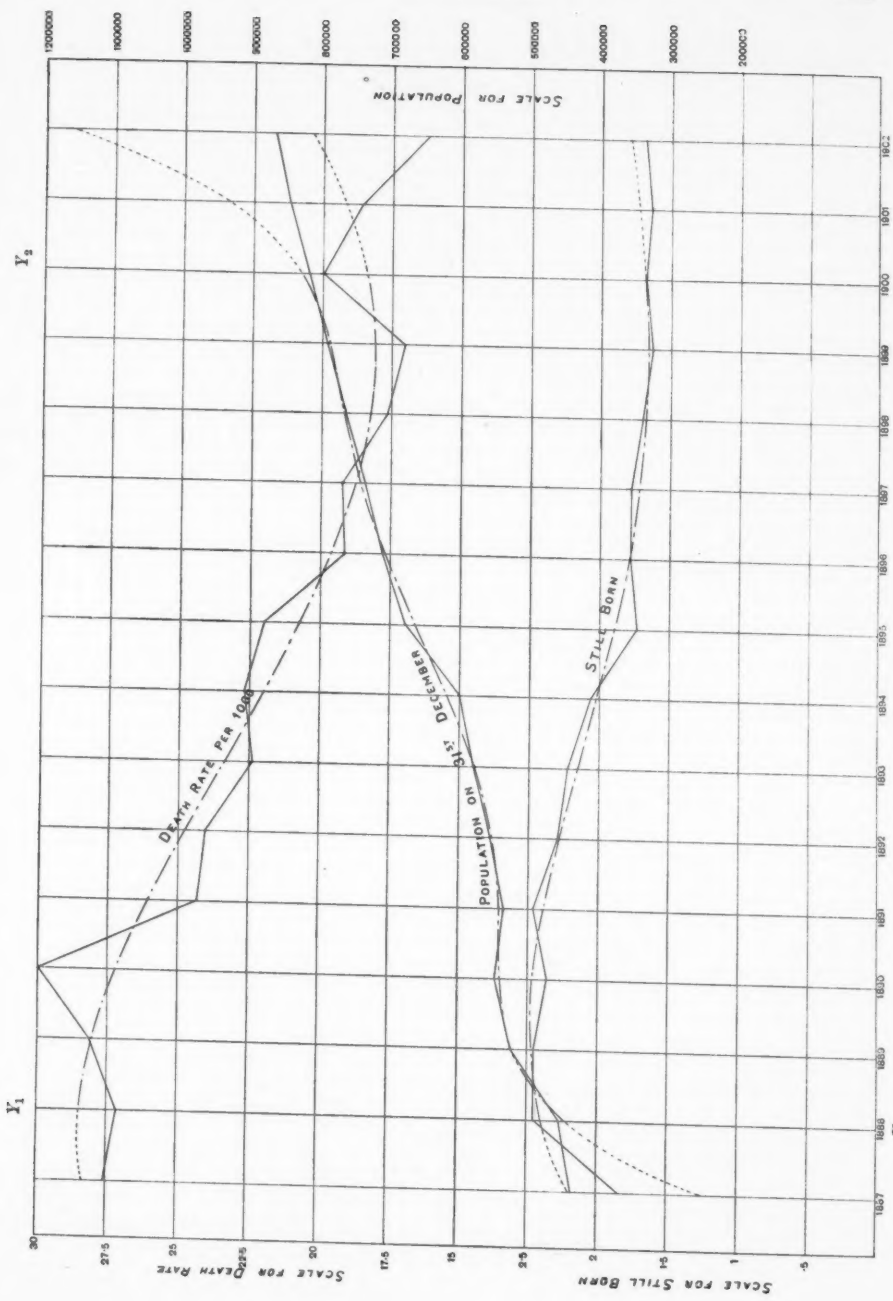


Diagram illustrating Danger of Extrapolation.

II. On Differentiation and Homotyposis in the Leaves of *Fagus sylvatica*.

By KARL PEARSON, F.R.S., with the assistance of MARION RADFORD.

(i) If an examination be made of a beech tree there will be found to be a distinction between the nature of the growth at the free higher parts of the tree and at the lower more accessible portions. Botanists have distinguished these two types as 'sun' and 'shade' branches. How far these differences of growth are entirely due to differences merely of light or even of air environment may be legitimately doubted. The free south side of a large beech tree does not as a rule abound in sun branches, while the north side exhibits only shade branches. There appears to be not only some factor beyond the light and air environment controlling the development of the tree, but the age of the tree and the age of the part of the tree considered seem also influential. Sun branches at any rate occur where the growth of the tree is most rapid, and shade branches where it is less rapid. Our experience is that with old and large beech trees, even if they are growing freely on a common, the sun branch is confined to the upper portions of the tree, and a random collection of sprays will result in shade branches only. The accompanying photographic reproductions which I owe to the kindness of Professors F. W. Oliver and A. G. Tansley will illustrate the difference between the two kinds of growth. We have collected a considerable number of sprays from beech trees this autumn and examined a considerable number collected by other persons, and it is safe to assert that a random gathering round the accessible parts of a large beech tree will result in a collection consisting in great bulk or more probably entirely of 'shade' branches.

(ii) If a spray representing a year's growth be cut from a shade branch of a beech tree, there will according to our experience generally be found to be four, occasionally only three, more rarely five leaves upon it. The accompanying Plate II. represents a number of such sprays, the typical forms being I. and II. Looked at from the back we shall call the leaves that fall on the right of the stem of the spray, right-hand leaves, and on the left of it, left-hand leaves. Measuring from the top of the spray down it, the leaves will be called first, second, third, etc., leaves. The leaves are almost invariably—in 99 per cent. of the cases observed—on alternate sides of the stem. 206 such sprays were gathered from the beech trees growing on Highmore Common, not far from Nettlebed on the Chilterns, by Dr Alice Lee and myself—the bulk from very large forest trees. Of these sprays 98, instead of 103 the half, had the first leaf on the right-hand side. In one case the third leaf was the first on the right-hand side; in the remaining 107 cases the second leaf was the first on the right-hand side. On the left-hand side there were 108 cases of first leaf and 98 cases of second leaf. We think we may therefore assert that the first leaf is as likely to be on the right as on the left side of the stem of the spray. The first and second leaves of the spray are both much larger than the third and fourth leaves, and there is a marked differentiation between the first pair and the second pair. This differentiation is so obvious, the second pair being in many cases quite dwarfs, that for the first paper* dealing with beech leaves we did not collect this secondary pair, and such pairs were therefore not included in the countings of the veins. The object of the present note is to test (i) whether serious error was introduced by disregarding the differentiation of the members of the first pair, and (ii) what effect the inclusion of the secondary leaves would have upon the homotyposis.

(iii) Investigating first the difference between the first leaf on the right and the first on the left without regard to order, and dealing with the same character as in the earlier investigation, i.e. the total number of veins on both sides of the main ridge of the leaf, we find :

	Mean	Standard Deviation
First Left-hand Leaf...	15·432 ± ·075	1·589 ± ·053
First Right-hand Leaf	15·495 ± ·074	1·585 ± ·053

* "Homotyposis in the Vegetable Kingdom." *Phil. Trans.* Vol. 197, A, p. 324.

There is clearly then no sensible differentiation between the number of veins or its variability when we pass from the right to the left-hand side of the stem.

Next dealing with the leaves taken in order down the stem, and without regard to their right or left-handed position, we have:

	Number	Mean	Standard Deviation
First Leaf ...	206	15·684 ± ·071	1·505 ± ·050
Second Leaf ...	206	15·257 ± ·076	1·612 ± ·054
Third Leaf ...	205	13·098 ± ·096	2·034 ± ·068
Fourth Leaf ...	145	10·559 ± ·110	1·968 ± ·078
Fifth Leaf ...	23	8·348 ± ·204	1·448 ± ·144

The mean of all the first and second leaves together may be compared with the Great Hampden series of 1900, also from the Chilterns:

	Mean	Standard Deviation
Highmore Beeches ...	15·471	1·574
Great Hampden Beeches	16·106	1·735

Considering the difference of localities, of season and class of tree, these results appear to be in reasonably close agreement, and enable us to judge how far any serious error was introduced into the Hampden results by the hypothesis that the large, i.e. first or second leaves on the spray, were undifferentiated. The Hampden leaves have clearly more veins, there is a sensible difference of ·6 in the mean, not very great, but sufficient to show that we had properly excluded the third and fourth leaves, when we collected. In the next place there is a sensible differentiation as we go down the stem, most marked when we pass from the first two leaves to the third, fourth and fifth, but hardly of great importance when we consider only the first and second.

If we treat as we did in the Hampden series the first two leaves as homotypes and work out their correlation we find:

Homotypic Correlation of first two leaves

$$\rho = \cdot 504 \pm \cdot 025.$$

Now let us work this out, allowing for the differentiation between the two leaves. Using the formula given in *R. S. Proc.* Vol. 71, p. 302, i.e.:

$$r = \rho \frac{\sigma^2}{\sigma^2 - \sigma_M^2} + \frac{\sigma_M^2}{(m-1)(\sigma^2 - \sigma_M^2)},$$

we have:

$$\rho = \cdot 5042,$$

$$\sigma^2 = 2 \cdot 47731, \quad \sigma_M^2 = \cdot 045582$$

$$\sigma^2 - \sigma_M^2 = 2 \cdot 34839. \quad m = 2.$$

Whence we find:

$$r = \cdot 5513.$$

The actual value given for homotyposis in beech leaves of the Hampden series being ·5699, it will be seen that only a small change has been made by allowing for differentiation in the present results from the Hampden value. The simple explanation of this undoubtedly being that our leaves were almost entirely, owing to our method of gathering, the *first* and not the second leaves of the year's growth.

But the present investigation allows us to go a stage further, and to find the homotyposis in beech leaves supposing we take all leaves and not merely the two first leaves of the year's growth. We have 144 sprays* with four leaves on, five leaves are so rare that we cannot use

* In the case of the 145 sprays with fourth leaves recorded, there was one in which a third leaf was not available for counting.

the fifth leaf. Let us find the homotyposis of the series as a whole, correcting for differentiation.

We have: $\rho = \cdot0300 \pm \cdot0162$.

In other words, when we do not pay attention to differentiation the homotyposis is almost zero. Clearly the small leaves on the spray could not be disregarded even by a most careless gatherer, without the result being very manifest when the correlation is determined.

Now let us allow for this gross case of differentiation by the above formula. We have

$$\rho = \cdot0300 \pm \cdot0162,$$
$$\sigma^2 = 7\cdot28052, \quad \sigma_M{}^2 = 3\cdot88274,$$
$$\sigma^2 - \sigma_M{}^2 = 3\cdot39778. \quad m = 4.$$

Hence: $$r = \cdot0643 + \cdot3809$$
$$= \cdot4452.$$

Considering the comparative paucity of the material—only 144 sprays—I think this result may be considered fairly satisfactory. Namely, we find, that judged by the first two leaves the homotyposis is ·55, agreeing closely with the earlier determination; judged by the first four leaves of the spray the homotyposis is ·45. The probable error of either result is certainly not less than ·04 to ·05*, and we conclude that the homotyposis in beech leaves cannot diverge much from ·5. Experience seems to show that the wider range of homotypes taken, if we allow for differentiation, the better the result will be. The present paper is also of interest as showing that a slight differentiation such as that between the first and second leaves, even if it should escape detection, would not radically modify, still less vitiate the results obtained. To get final numbers probably 1000 sprays of four leaves ought at least to be dealt with. The present study, however, will indicate how the student of homotyposis can investigate and allow for differentiation due to position. Other instances are given in the memoir in the *R. S. Proceedings*, Vol. 71, pp. 288–313.

(iv) A paper has been recently published by Miss Tine Tammes† in which two tables are given for the number of veins on beech leaves, having regard to their position on the year's shoot (*Jahrestriebe*). The authoress gathered 15 shoots with 9 leaves and 6 shoots with 8 leaves, —all off the *same* tree. She sought shoots with the same number of leaves as they had internodes: "Diese waren, obgleich die Zahle der Internodien bei dem verschiedenen Trieben ziemlich stark variirt, an dem grossen Baume ohne viel Mühe zu finden" (p. 78). Further the shoots taken were those which had the greatest number of internodes.

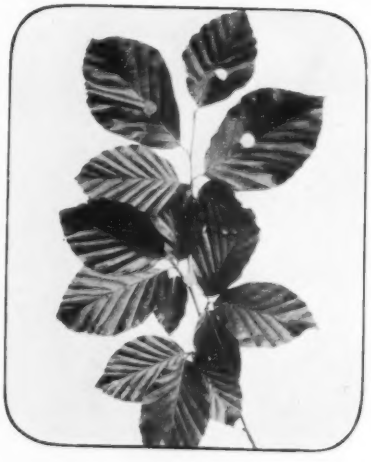
She found for this single tree:

Position of leaf from base of shoot	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Mean No. of veins on 15 shoots	9·8	11·7	13·1	13·5	13·1	12·7	11·9	10·6	9·5
„ „ 6 shoots	10·0	11·7	13·3	13·8	13·7	12·8	12·0	11·0	—

Now what is quite clear from this table is that (i) the nature of the shoots dealt with by Miss Tammes is both for number of leaves and number of veins to the leaf entirely different to what would be obtained by a random collection of year's shoots on the Chiltern beeches. That (ii) had she made such a random collection, her law of periodicity in growth would not have received confirmation from the beech tree at all.

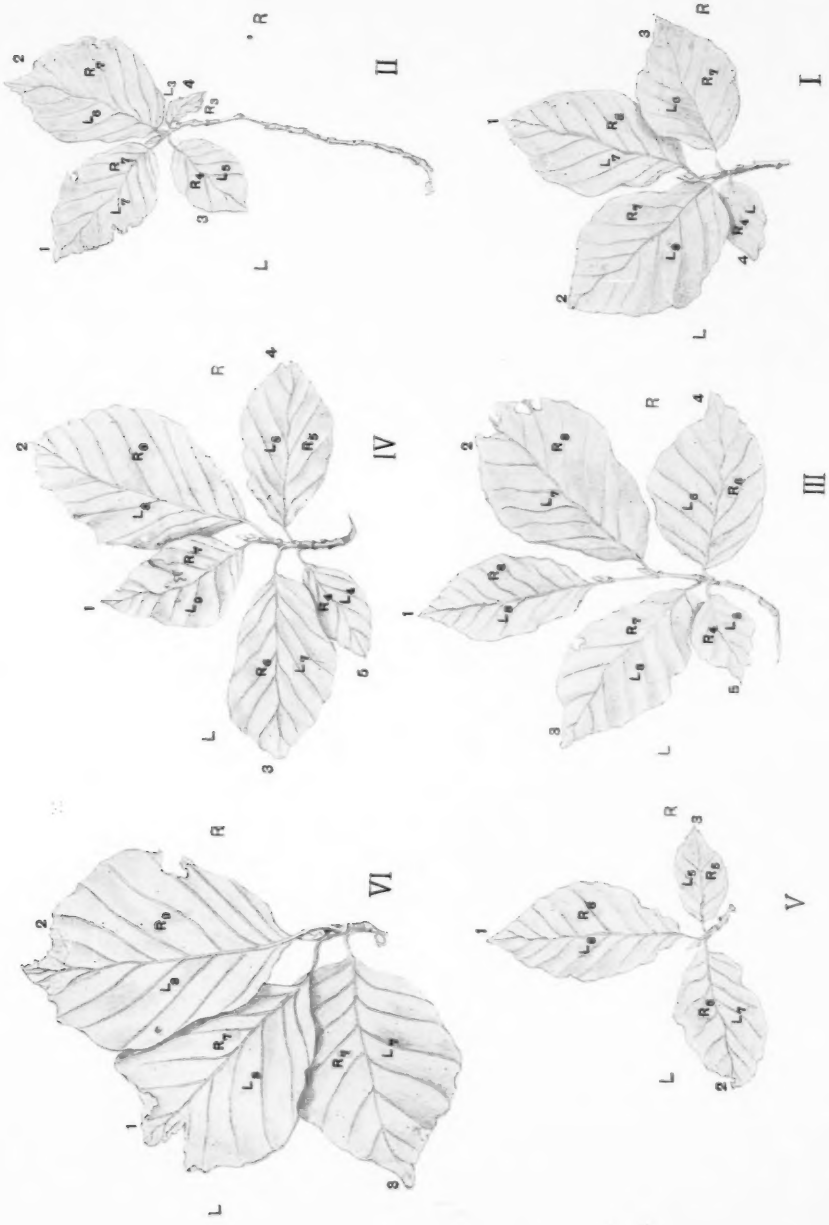
* For example, with a deviation of ρ from ·03 equal only to about its probable error, the first term in r will be raised to ·10 instead of ·06 and thus r would be sensibly ·5.

† *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam* (Tweede Sectie), Deel IX. Die Periodicität morphologischer Erscheinungen bei den Pflanzen, pp. 1—148. Maart, 1903.



'Shade' Branch.

'Sun' Branch.


I
II
III
IV
V
VI

There seems therefore considerable danger both in drawing conclusions from *selected* shoots of a *single* tree in a problem of growth and further in citing such results as an argument as to homotyposis. Probably, but we have no certainty for it, Miss Tammes in selecting shoots with 9 leaves, really made a selection of 'sun' branches as distinguished from 'shade' branches, but whether her results are universally true for the leaves on all such branches of all beech trees, it is quite impossible to say until a large number have been examined. In the case of the big trees such as we have dealt with from Hampden and Highmore, the task of collecting year's shoots from 'sun' branches will not be an easy one, for in the majority of cases they are very inaccessible. Whether the homotyposis of 'sun' branches differs from that of 'shade' branches, it is impossible to say; the larger number of leaves to the shoot on the former, however, would make them better material for homotypic investigation.

(v) It may perhaps not be out of place to note here that since the first memoir on homotyposis was written we have been able to get a short series of 150 woodruff sprays from a lane at Horsham, Sussex. The woodruff was in flower and the whorls were counted from the flower downwards. As compared with the Great Hampden* series we found:

	Mean number of branches	Standard Deviation
Great Hampden ...	6·9	·93
Horsham	6·7	·84

The correlation between position and number of branches was ·26 and the homotyposis allowing for this differentiation was ·32. This is a very sensible increase on the ·17 of the original Hampden investigation, but while the number of whorls to the spray at Hampden was on an average 7 and might reach 10, the mean number at Horsham was 4·5 and never exceeded 5. Hence the material did not provide a good determination of the relationship between number of branches and position of the whorl. What it does suffice to show is that the Hampden series was justly described as having a value much reduced by neglect of differentiation. We should be very glad if any reader of *Biometrika* would provide us with 400 to 500 healthy sprays of woodruff after the plant has ceased to grow in the autumn of next year.

Description of Plates.

Plate I. Illustrations of the so-called 'sun' and 'shade' branches of the beech.

Plate II. Backs of typical beech leaf sprays. I and II. Usual forms of spray, with four leaves and indicating the marked differentiation between first and second pairs of leaves. II and III. Seldom forms with five leaves. V and VI. Less seldom forms with three leaves only. The letters *L* and *R* indicate the left and right-hand side of the back of the leaf and the subscript figures the number of the veins. The larger number gives the order of the leaf on the spray stem.

III. Albinism in Sicily and Mendel's Laws.

It has been suggested by Castle (1) that albinism in Man behaves in accordance with Mendel's Laws of Inheritance, and the suggestion is considered probable by Bateson (2). In a recent paper Garrod (3) has endorsed the same view, and has cited the work of Arcoleo in support of it. Arcoleo's work (4) records 62 cases of albinism observed in Sicily, the greater number in the province of Palermo. Three of the albinos recorded were produced by albino parents, the rest had normal parents. In ten cases it is known that the parents were pigmented, but it is not known whether they produced pigmented children in addition to the albinos.

The forty-nine remaining cases were distributed among twenty families, which contained altogether 133 children. Now, on the Mendelian hypothesis, that albinism and possession of pigment

* The Great Hampden Lane gave no woodruff when examined this autumn, but the hedges had been recently cut down and the banks trimmed.

are allelomorphic characters, albinism being recessive, the marriages which can give rise to albino children are of two kinds only, namely (1) those between albinos of any ancestry, who are by the hypothesis "gametically pure recessives," and (2) those in which each individual has some albino ancestry, and is a "heterozygous" product. Marriages of the first kind should produce albinos only, while a quarter of the children resulting from marriages of the second kind should be albino, the rest being pigmented. Neither of these expectations is justified by Arcoleo's record. The three albinos produced by albino parents belonged to one family, and it is stated that "Gli antenati furono *tutti* bianchissimi," implying a considerable amount of albino ancestry; yet this family contained two pigmented brothers! Again, the most probable number of albinos among the 133 offspring of pigmented parents, all of whom are known to be capable of producing *some* albinos, is 33·25, and the Standard Deviation of this expectation is $\sqrt{\frac{1}{4} \times \frac{3}{4} \times 133} = 4{\cdot}94$. The observed number of albinos, namely 49, shows an excess of 15·75, or about 3·2 times the standard deviation, over that predicted by the Mendelian hypothesis. The odds against any result as large or larger than that observed, if the hypothesis be true, are nearly 2000 to 1; and although this degree of improbability may not be held by some to disprove the hypothesis, it certainly ought not to be adduced as evidence in support of it.

The occurrence of segregation in a non-Mendelian proportion is exactly paralleled by the behaviour of the character "waltzing" as opposed to normal walking in mice, recorded by von Guaita and Darbishire.

The one fact which is not in apparent contradiction to Mendel's hypothesis is the behaviour of six albinos who married pigmented persons. One marriage was sterile; the other five produced altogether 24 children, all pigmented.

Taking these records in conjunction with the work of Cuénot (6) and Darbishire (5) on the inheritance of albinism and of other characters in mice, we see that the disappearance of a character such as albinism in the offspring resulting from a cross between an albino and a pigmented individual, and the reappearance of albinism in a portion of the grandchildren, produced when the immediate offspring of the cross are mated, may be associated with very different conditions of the albinos. Cuénot and Darbishire have both shown that the colour of the young produced by a cross-bred albino mouse, paired with a coloured mouse, depends partly on the colours exhibited by the ancestors of the cross-bred albino, thus showing clearly that such an albino, although it should be a "pure recessive" in Mendel's sense, is not gametically pure. Among mice there is no certain record of cross-bred albinos so impure that when paired together they produced pigmented young; the case of the family recorded by Arcoleo is therefore of great interest, as showing that in Man even this degree of "gametic impurity" may exist in albinos.

These results show how necessary it is that the phenomena of alternative inheritance should be studied in the light of fuller experimental knowledge concerning the correlations between cross-bred individuals and their ancestors; they show the futility of attempting to express such phenomena in terms of formulae based on the unproved hypothesis of gametic purity.

Finally I would urge on those who have opportunity the great value of a full study of albinism in Sicily. In the province of Palermo, from which many of Arcoleo's cases are drawn, the percentage of accepted soldiers of pure blonde type ("con capelli biondi e con occhii celesti") is given by Livi (7) as 2·1; that of men "con capelli biondi o rossi e con occhii celesti o grigi" is 3·8. Albinos are naturally exempt from military service, because of their abnormal eyes, and therefore do not appear in Livi's table. During a recent visit to Sicily, when I walked over a good deal of country round Palermo with an artist friend, my attention was continually called, not only to albinos, but to the extraordinary fairness of many blonde persons, who were still slightly pigmented. It would be of great interest to know whether these blondes are more often related to albinos than are persons of the darker, more usual type.

W. F. R. W.

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6. Cuénot. L'Hérédité de la Pigmentation chez les Souris. *Arch. Zool. Exp.* (Notes et Revue), 1902. Sér. 3, T. x. No. 2.
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IV. A Mendelian's View of the Law of Ancestral Inheritance.

It is a very thankless task to try and correct every mis-statement that untrained minds make when they attempt to deal with a statistical problem. But one can only hope that the persistent exposure of the blunders made by non-statistically trained biologists, when they treat problems of heredity, may ultimately produce some effect. To the statistician nothing is more obvious and intelligible than the independence of the correlations, variations and means of characters. A knowledge of any one of the three involves no knowledge of the other two. Again, equally distinct in his mind are the multiple regression coefficients and the correlations. No practised biometrician could for a moment confuse with the ancestral correlations the multiple regression coefficients, which appear when we calculate the most probable deviation of an individual from his own type with the known deviations of his ancestors from their types; still less would he confuse a correlation coefficient with the number of offspring that take after a parent or grandparent! It is difficult to understand at all the attitude with which a biologist like Professor Castle* approaches a problem in inheritance, but it is summed up in the words that he has not the least idea that there is any distinction between a correlation, a multiple regression coefficient, and the number of individuals who may take after an individual ancestor!

Mr Francis Galton published in his *Natural Inheritance* (p. 136), "with hesitation" the following statement "consequently the influence of the individual parent would be $\frac{1}{4}$, and of the individual grandparent $\frac{1}{16}$ and so on."

In his work on Bassett Hounds, Mr Galton subsequently extended this principle by supposing that of the "heritage" as represented by all the offspring together, $\frac{1}{4}$ of the total would on the average have the character of each parent, $\frac{1}{16}$ that of each grandparent, $\frac{1}{32}$ that of each great grandparent, and so on. This statement is obviously different from that in *Natural Inheritance*, where Mr Galton is treating of a blending character. It deals with alternative inheritance. From these numbers Professor Castle deduces "Galton's Series." He calls the parental influence ·50, the grandparental influence ·25, the great grandparental ·125 and so on. I presume that he understands by this the proportions in the total offspring who will be like each individual ancestor. Against these proportions Professor Castle puts a series which he calls "Pearson's

* "The Laws of Heredity of Galton and Mendel, and some Laws governing Race Improvement by Selection," by W. E. Castle. *Proceedings of the American Academy of Arts and Sciences*, Vol. xxxix, pp. 223—242.

Series," in which he has taken the correlation coefficients with a *single* parent or a *single* grandparent etc., as I have deduced them for man and horse as if they were quantities in the least comparable with the number of individual offspring. He remarks:

"Comparing Pearson's series with that of Galton, we see that the parental influence is reckoned as substantially the same by both Galton and Pearson, but that Pearson assigns a much greater influence to the more remote ancestors than does Galton*."

Had Professor Castle merely added up the numbers he gives in this "Pearson's Series" he would have realised that they could not possibly represent what he states them to do; for the four grandparents alone would have had more offspring "like" themselves than the total number of offspring!

Now the Law of Ancestral Heredity as stated by me gives absolutely no means of ascertaining the number of offspring "like any given ancestor"; and if we measure parental influence by intensity of *correlation*, we should have:

	Pearson's Series	Galton's Series
Parental Influence	1/2	1/3
Grandparental Influence	1/3	1/9
Great grandparental Influence	2/9	1/27
Great great grandparental Influence ...	4/27	1/81

whence the complete misrepresentation of Professor Castle will be obvious.

After calculating a table which he says gives the distribution of the offspring of a cross between grey and white mice for 7 generations on Galton's Law, Professor Castle continues:

"The observed numbers, it is evident, agree no better with one of Pearson's series than with that of Galton. The discrepancies noted between observed and calculated will remain and even be accentuated if we replace Galton's series with one of those† suggested by Pearson. For the result will be unchanged in generation II, but the calculated numbers will in most cases diverge still more from the observed ones, in the later generations, because Pearson attaches more weight to the remoter ancestors than does Galton."

Now I suppose Professor Castle had some idea in his mind when he penned these lines, but so far as I can understand the Law of Ancestral Heredity as I have myself enunciated it, the produce of a grey mouse and a fawn mouse might be on the average a green mouse without that Law having anything to say on the point. From it you cannot possibly deduce what number of the offspring of any generation will be like this or that ancestor. It is not a law of types, but of the distribution of deviations from type, and this is a very different thing indeed.

The Law of Ancestral Heredity, as I have repeatedly stated‡, makes no assumption as to the mean character of any generation; it gives no statement whatever as to the number of offspring who are like any individual ancestor. What it does give us is this: the means of determining the probable deviation of any individual from the *type of his own generation*, when we know the deviations of some or all his ancestry from the types of their respective generations. What then is the simple relation of my Law of Ancestral Heredity to Galton's series of $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{16}$ etc.? It lies in the following hypotheses: (i) Galton supposes the type of each generation to remain the same, and

* p. 224, *loc. cit.*

† Professor Castle quotes the two series I have given in *Biometrika*, Vol. II., p. 222, the one as the best geometrical series, and the other as a close series, in which round numbers were taken.

‡ *R. S. Proc.* Vol. 62, pp. 387 *et seq.*, where all the deviations are expressly said to be measured from the type of each generation, so that any modification of the type may be allowed for. Also in *Biometrika*, Vol. II., p. 217, where we are again told that the deviation is from the type of its generation, and are expressly warned (p. 226) that no assumption is made that the mean of the forecasted character is identical or not with any of the means of the foreknown characters. These papers are actually cited by Professor Castle.

expressly states that he is *dealing with a stable population*. Notwithstanding Galton's own statement as to his series holding for a stable population, Professor Castle does not hesitate to apply it to a cross which produces a totally different population! My types may alter and do alter in any way from one generation to a second. (ii) Galton supposed that we may apply his series not only to the deviation of an individual from type, but to the "whole heritage," he divides the offspring up into groups of individuals following special ancestors. This is an extension I have expressly and repeatedly disagreed with. I look upon such a distribution as not falling under the Law of Ancestral Heredity at all, but as part of the Theory of Reversion or of Alternative Inheritance. My agreement with Galton consists in the general conceptions (*a*) that the deviation of an individual from the type of his own generation depends ultimately on the deviations of his particular ancestors from their types, and (*b*) that the proportions of such deviation from type contributed by each generation of ancestry diminish in a geometrical series. This series is not Galton's. It may be deduced from the ancestral correlations, and in the case of man, horse and dog the series are within the limits of errors of observation identical. It is these ancestral correlations (which have no relation whatever to type, and are only connected with the regression coefficients which give the proportions of the deviation by complex determinantal relations) which Professor Castle cites, and states do not agree with the numbers of coloured mice that von Guaita found descended from albino-grey crosses in successive generations!

Now Professor Castle was perfectly free to ignore my work, or to confess frankly that he could not understand it, but he commits a grave breach of scientific decorum when in a paper, not taking hasty journalistic form but published by the American Academy, he states that "the test of von Guaita's mice is conclusively in favour of Mendel's Law and against the law of ancestral heredity," and yet shows that he has either not read, or not been capable of properly citing, the two papers in which the meaning of this law is discussed. Either Professor Castle is so ignorant that he does not know that a coefficient of correlation cannot be a group frequency; or, he has directly misquoted my memoirs because any form of argument suffices for the audience he wishes to appeal to. It is a fundamental canon of scientific discussion that, if you wish to contradict a man's results, you should know what those results are and cite them correctly. It is a breach of scientific decorum to assert that a man's theory is opposed to certain facts, when you have demonstrably not the faintest notion of what that man's theory is, or how his results are reached.

I stated in my paper of 1903 on the Law of Ancestral Heredity that "as far as the available data at present go, inheritance coefficients for ascending ancestry are within the limits of observational error represented by a geometrical series and by the same series." Professor Castle remarks:

"It should be observed that the 'available data' upon which principally Pearson bases his conclusions consist of two cases of pigment inheritance, one in man, the other in the horse. A third well-known series of this sort has not been utilized by Pearson, though our information about it is much more complete and precise than that about either of the other two. I refer to our statistics about colour inheritance in mice recorded by von Guaita*."

Now there are two points to be considered here. Professor Castle states (i) that the information is far more complete and precise, and (ii) that it is available for discussion by aid of the Law of Ancestral Heredity. Any one who has compared the total number of original parents or of offspring in any generation in von Guaita's statistics will realise that they are absolutely incomparable with the numbers we possess for man, horse or hound. They are also statistically insufficient to give correlation coefficients significant having regard to their probable errors. Further, the data given, as in almost all Mendelian categories, are so wanting in

* Castle: *loc. cit.*, p. 224.

precision that it is impossible to measure the deviations from the mean of any generation and so obtain the correlations. Not until far more precise classifications are made than von Guaita provides would the material be "available."

In the next place, to what material can the ancestral law at present be applied? It was perfectly open to Professor Castle to examine the conditions under which its results so far have been reached. They are stated with perfect precision in my memoir of 1898, and are (i) the absence of assortative mating, (ii) the absence of in-breeding and of selection. Now in von Guaita's experiments he got mice in which the ancestry had been assortatively mated for generations: he first put like to like or made a perfect coefficient of assortative mating; and then he crossed like with unlike, or made the coefficient of assortative mating negatively perfect.

This coefficient is taken *zero* throughout my memoir on the law of ancestral heredity, and the reader is told that the author hopes to deal in a later paper with the influence of assortative mating and in- and in-breeding. Notwithstanding this direct warning Professor Castle does not hesitate to speak of the Law of Ancestral Heredity as applicable to von Guaita's data, and while the propounder of that law has not yet been able to master the difficulties of the analysis which arises when intense assortative mating and generations of in-breeding are taken into account, the biologist remarks that the observed numbers, "it is evident," disagree with Pearson's series. Had I been able to master the analysis, von Guaita's data would not have provided the material upon which the correlations could be based, and so the law itself tested; they are simply not "complete and precise" enough.

Professor Castle, as I have already stated, reaches his result by taking my correlation coefficients for inheritance in man and horse, and asserting that they give the proportions of offspring who will be "like the ancestors." As I have before indicated, the typical ancestor might be blue and the typical offspring yellow without this having any bearing at all on the correlation coefficients. Personally I have no means of determining whether the law of ancestral heredity holds or does not hold for coat colour in mice. The theory has not yet been worked out in a form covering von Guaita's cases, and data are only at present being collected complete and precise enough for the purpose.

In the face of these facts I directly challenge Professor Castle to confess that he did not when writing his paper know what a coefficient of correlation was, and that he was thus incompetent to discuss the application of the law of ancestral heredity to mice; or else to show that either the correlation coefficients, or, if he likes better, the multiple regression coefficients, which lie hidden in von Guaita's data can really be extracted therefrom and are inconsistent with the values which I have found for man, horse and dog, when allowance is made for assortative mating and selection. I do not see that any other course is open to Professor Castle, after his statements that "it is evident" that the numbers do not fit my series and that "it is evident" that some fundamental defect exists in the "law of ancestral heredity." It is time that some check should be administered to this irresponsible and ignorant criticism of biometric methods, a criticism which can confuse a group-frequency—i.e. a number of individuals —with a correlation coefficient, a relation between deviations from type in individuals; and I therefore directly challenge Professor Castle to justify his statements, or failing that I call upon him to retract them.

These statements are as follows:

(*a*) Pearson's Law of Ancestral Heredity can be applied to test von Guaita's data for mice, and,

(*b*) When so applied, "it is evident" that it does not fit them.

K. P.